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MODELLING 'DON'T KNOW' RESPONSES IN RATING SCALES

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INTRODUCTION

Survey questionnaires aimed at investigating human perceptions often contain **rating** scale questions with the '**don't know**' option (*dk*)

In the literature, *dk* responses have been studied extensively: whether to include the dk option or not and the statistical treatment of dk responses are open issues

Statistical models are usually unable to properly account for *dk* responses (which often ends up being treated as a missing value)

dk is a **valid response** to all extents: it contains important information about the **uncertainty** of the subject in formulating the response

AIM

To propose a probabilistic framework for the treatment of *dk* responses in rating scales

Our proposal finds an insightful definition within the CUB models

OUTLINE

- CUB models
- A statistical model for the treatment of *dk* responses
- Case study

CUB MODELS (Piccolo, 2003; D'Elia & Piccolo, 2005)

Introduced in the literature to analyze rating (or ranking) data

Each subject's rating is interpreted as the combination of

- a **feeling** attitude towards the item being evaluated
- an intrinsic **uncertainty** related to the discrete choice

These components are considered in the CUB models by a **mixture** of a Discrete Uniform and a Shifted Binomial random variables:

$$Pr(R=r;\theta) = \pi \boxed{b_r(\xi)} + (1-\pi) \boxed{Pr(U=r)}$$

with $r = 1, 2, \ldots, m,\ \theta = (\xi, \pi)',\ \xi \in [0,1],\ \pi \in (0,1]$

CUB MODELS

Parameters are related to the latent components of the responses:

$1-\xi$ → **feeling** with the item

$1-\pi$ → **uncertainty** of the choice

There is a one-to-one correspondence between a CUB and $\theta = (\xi, \pi)'$

→ each CUB model can be represented as a **point** in the unit square (with coordinates $1-\pi, 1-\xi$)

Among several fitting measures, we consider $Diss = \frac{1}{2}\sum_{r=1}^{m}\left|\frac{n_r}{n} - p_r(\hat{\theta})\right|$

THE PROBABILISTIC FRAMEWORK TO HANDLE *dk*

A latent trait Y is evaluated by a random sample of n subjects asked to express a rating R on a given ordinal scale from 1 to m

$Pr(R = r; \theta)$ depends on $\theta = (\xi', \pi')'$ where

- ξ is related to the level of Y (**feeling** parameter)
- π is independent on Y but together with ξ influences the probability distribution of R; it accounts for indecision in the response (**uncertainty** parameter)

THE PROBABILISTIC FRAMEWORK TO HANDLE *dk*

We assume that the population is divided in 2 groups:

Respondents **able**

to express a rating

$A = 0$, with probability f

$P(R = r; \theta_0 | A = 0)$

$\theta_0 = (\xi_0', \pi_0')'$

Respondents **unable**

to express a rating

$A = 1$, with probability $(1 - f)$

$P(R = r; \theta_1 | A = 1)$

$\theta_1 = \pi_1$

Marginally, for a given subject i, we have

$$Pr(R = r_i; \theta) = f\,Pr(R = r_i; \theta_0 | A = 0) + (1 - f) Pr(R = r_i; \theta_1 | A = 1)$$

PARAMETER ESTIMATION

2 situations:

- *dk* option **not available**

  Information about A is missing

  → Specific statistical techniques (e.g., LCA)

- *dk* option **available**

  We estimate parameters under the assumption that subjects choose *dk* if and only if they belong to the $A = 1$ group

dk RESPONSE TREATMENT WITH CUB MODELS

The general proposal can be defined in several ways, depending on the choices made for

$$P(R = r; \theta_0 | A = 0) \qquad P(R = r; \theta_1 | A = 1)$$

These choices should ensure an interpretable formulation of the marginal distribution and the parameter θ

We choose

$P(R = r; \theta_0 | A = 0)$ — CUB model

$P(R = r | A = 1)$ — Discrete Uniform $U(1,\ldots,m)$

MOTIVATIONS FOR THE UNIFORM DISTRIBUTION

Among others, two main motivations are:

- It is **reasonable**: a person who answers *dk* considers any alternative equally agreeable (if he/she had some preference towards one rating, he/she would have chosen it)
- Evidence from the **literature on guesswork**: if subjects unable to express a rating were forced to give an answer, they would make a random choice among the response categories $(1,\dots,m)$

dk RESPONSE TREATMENT WITH CUB MODELS

Under the assumed conditional distributions, the marginal distribution is

$$Pr(R=r;\theta)=f[\pi_0 b_r(\xi_0)+(1-\pi_0)Pr(U=r)]+(1-f)Pr(U=r)$$

which can also be written as

$Pr(R=r;\theta,f)$ CUB model with $\theta=(f\,\pi_0,\xi_0)'$

The marginal distribution of the ratings accounting for the dk responses is a **CUB with higher uncertainty $1-\pi_{adj}=1-f\pi_0$**

PARAMETER ESTIMATION

It can be shown that parameters can be estimated separately:

- f is estimated by the **relative frequency of expressed ratings**
- The ML estimates (via the EM algorithm) of π_0 and ξ_0 are obtained by fitting a CUB model on the data with **listwise deletion** of *dk* responses
- π_{adj} is estimated by

$$\hat{\pi}_{adj} = \hat{f}\hat{\pi}_0$$

$1 - \hat{\xi}_0$ and $1 - \hat{\pi}_{adj}$ are the estimates of the feeling and uncertainty parameters **when accounting for *dk* responses**

GOODNESS-OF-FIT EVALUATION

- If we ignore the presence of *dk* responses, we can evaluate the goodness of fit of the CUB model to the observed frequencies of the **expressed ratings**, by means of a dissimilarity index $Diss_0$

- Otherwise, any goodness-of-fit index must rely on some **assumptions** about how the *dk* responses are partitioned among the response categories

GOODNESS-OF-FIT EVALUATION

It is possible to show that when the *dk* responses are partitioned among the response categories **according to the Uniform assumption**, the CUB model with adjusted uncertainty allows an **improvement** over $Diss_0$

This improvement (%) is equal to $(\hat{f} - 1) \cdot 100$

An improvement can be reached even with **deviations from the uniformity** assumption (the maximum admitted deviation from uniformity to improve $Diss_0$ depends on $Diss_0$)

CASE STUDY

Data come from **Standard Eurobarometer**, a sample survey covering the national population of citizens of the 27 European Union member states (the average number of interviewees over the 27 Countries is 986)

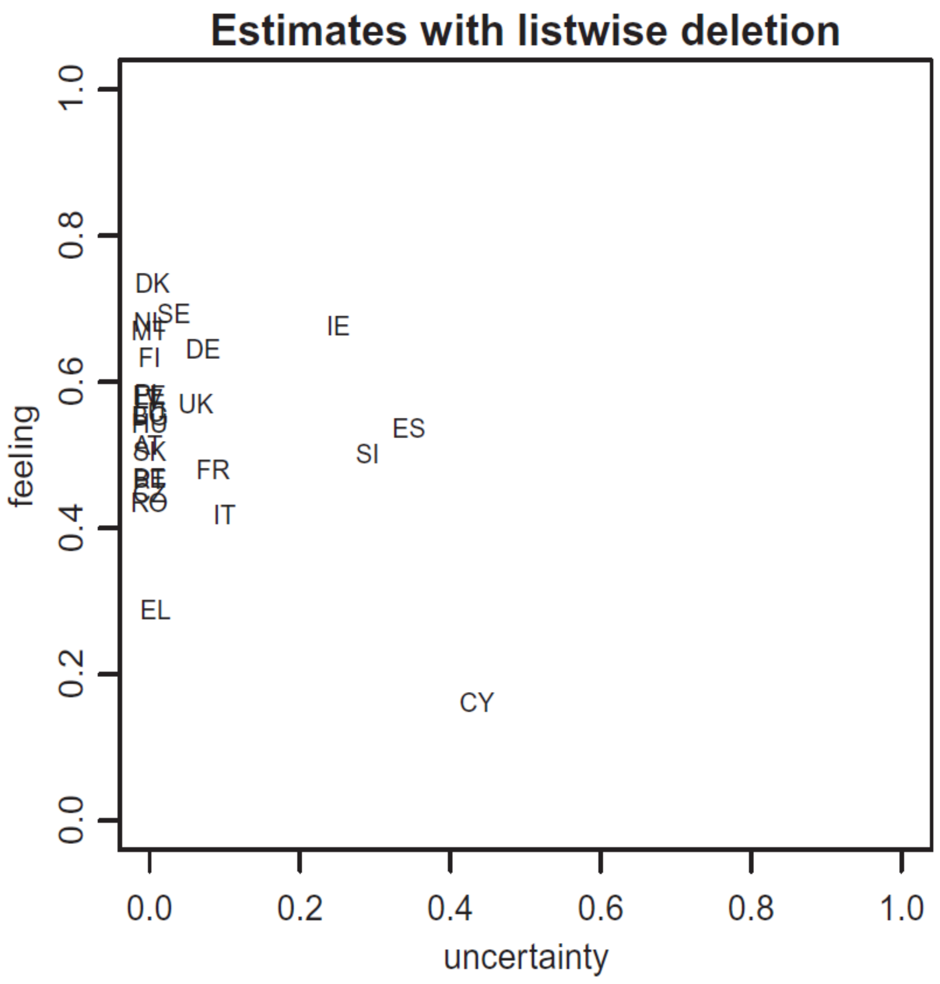
Focus on one question, measuring the agreement/disagreement with the statement "**Globalization is an opportunity for economic growth**"

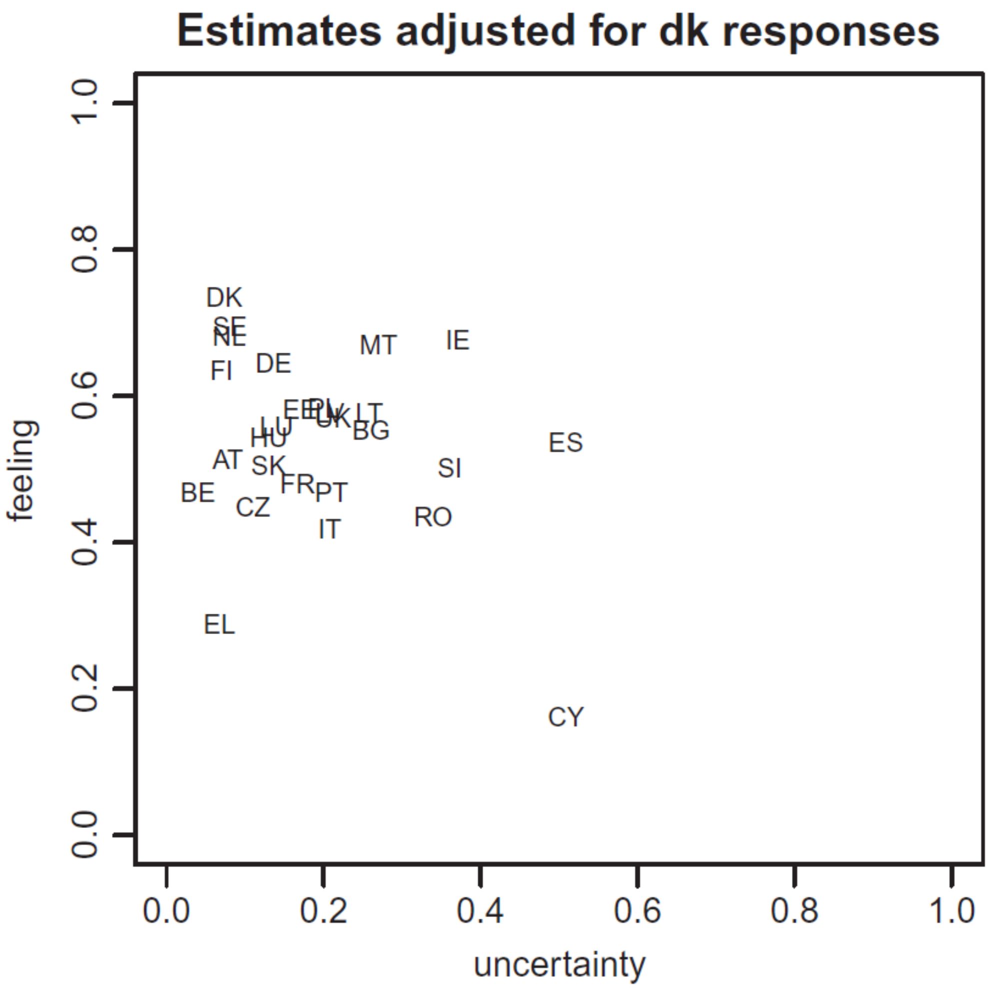
Response scale:

totally disagree - tend to disagree - tend to agree - totally agree - *dk*

% *dk* averaged over all countries is 14.67 (from 4 -Belgium to 34 -Romania)

CASE STUDY


Estimates with listwise deletion
feeling
uncertainty
0.0
0.2
0.4
0.6
0.8
1.0
DK
SE
MT
DE
FI
IE
UK
ES
SI
AT
SK
FR
RO
IT
EL
CY


Estimates adjusted for dk responses
feeling
uncertainty
0.0
0.2
0.4
0.6
0.8
1.0
DK
SE
NL
DE
FI
MT
IE
EE
PL
UK
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BE
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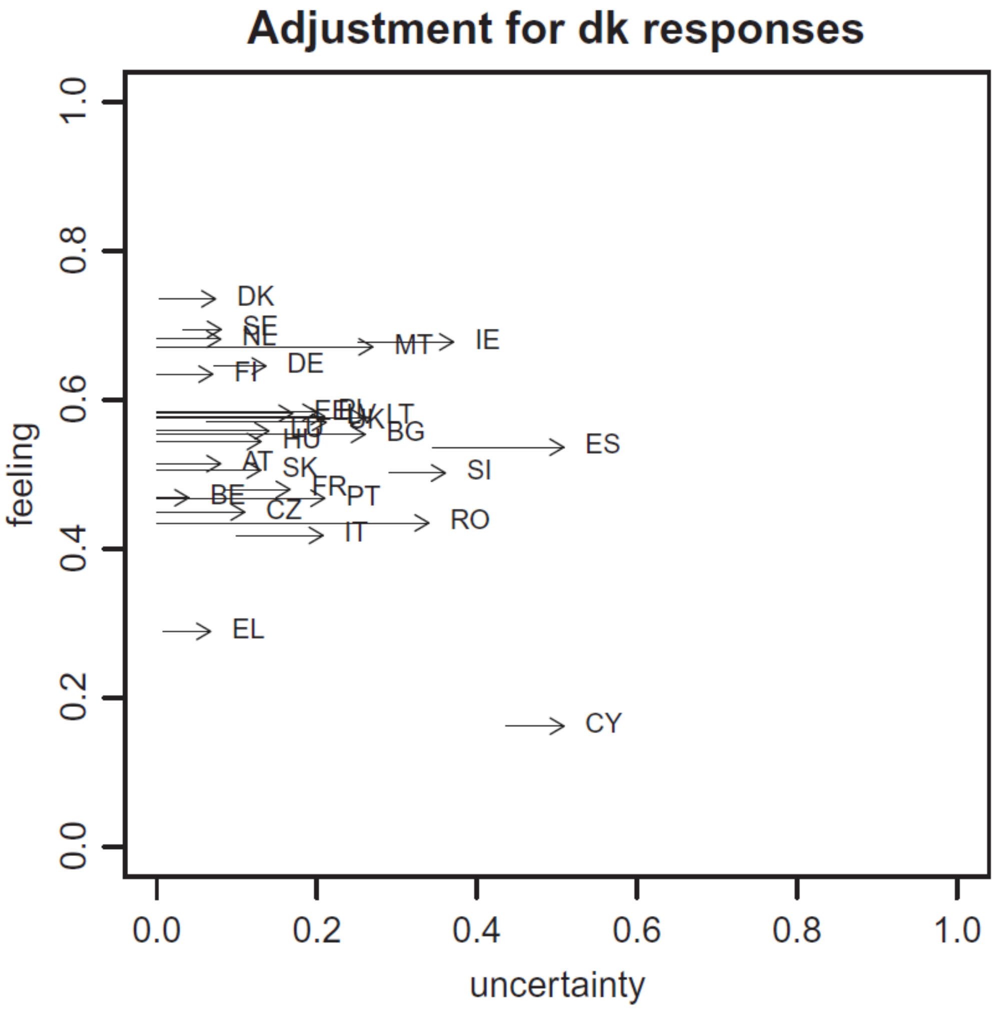
CASE STUDY



$Diss_0$ varies from 0.04 (Portugal-PT) to 0.16 (Malta-MT)

(average $Diss_0 = 0.09$)

Max decrement in $Diss_0$ when considering *dk* responses ranges from 4% (Belgium-BE) to 34% (Romania-RO)

CONCLUSIONS

We proposed accounting for *dk* responses by means of a statistical model containing a **feeling** parameter and a parameter that accounts for the indecision in the response (**uncertainty**)

Within the CUB framework, our proposal leads to a meaningful result, that is the **increase of the estimate of uncertainty** in the population, which is directly related to the estimated proportion of subjects unable to express a rating

We **do not replace** each *dk* with a substantive response

We treat the attitudes of respondents who 'don't know' at an **aggregate level**

Despite its simplicity, the proposed correction can be important when **comparing** feeling and uncertainty over several items or groups of people

BASIC REFERENCES

- D'Elia A. & Piccolo D. (2005). A mixture model for preference data analysis. Comput Stat Data An 49, 917-934.
- Manisera M. & Zuccolotto P. (2014). Modeling "don't know" responses in rating scales. Pattern Recogn Lett 45, 226-234.
- Piccolo D. (2003). On the moments of a mixture of Uniform and Shifted Binomial random variables. Quad Stat 5, 85-104.

THANK YOU FOR YOUR ATTENTION