Analysis of Rating Data in the CUB class framework

A brief overview of methods and models proposed by the BODAI – LAB research group

Paola Zuccolotto – Marica Manisera

Agenda

- The unconscious Decision Process (DP) driving individuals' responses on a rating scale
- CUB models (D'Elia&Piccolo 2005, Computational Statistics and Data Analysis)
- CUB models with DK responses (Manisera&Zuccolotto 2014, Pattern Recognition Letters)
- NLCUB model (Manisera&Zuccolotto 2014, Computational Statistics and Data Analysis) Econometrics and Statistics
- CUM model (Manisera&Zuccolotto 2022, Econometrics and Statistics)

➢ **With two real data examples (Likert scale + semantic differential scale)**

Rating data

The analysis of human perception is often carried out by resorting to **surveys** and **questionnaires**, where respondents are asked to **express ratings about the objects being evaluated.**

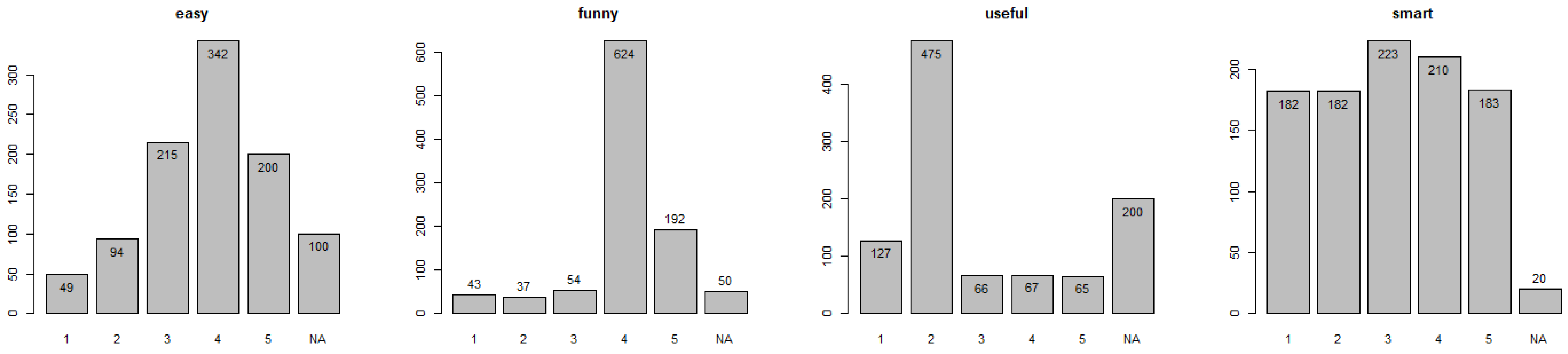
The goal of the statistical tools proposed for this kind of data is to explicitly **characterize the respondents' perceptions about a latent trait**, by taking into account, at the same time, the **ordinal categorical scale of measurement** of the involved statistical variables.

Rating data – example 1 (N=1000, 5-point Likert scale)

Please tell me to what extent you agree or disagree with each of the following statements (1=totally disagree, 2=tend to disagree, 3=nor disagree neither agree, 4=tend to agree, 5=totally agree, NA=don't know)

- This APP is easy to use, I learned very quickly all its features (**easy**)
- I really enjoy using this APP, it's kind of fun (**funny**)
- I find this APP very useful to my purposes (**useful**)
- This APP is very smart and cool (**smart**)

Rating data – example 1 (N=1000, 5-point Likert scale)



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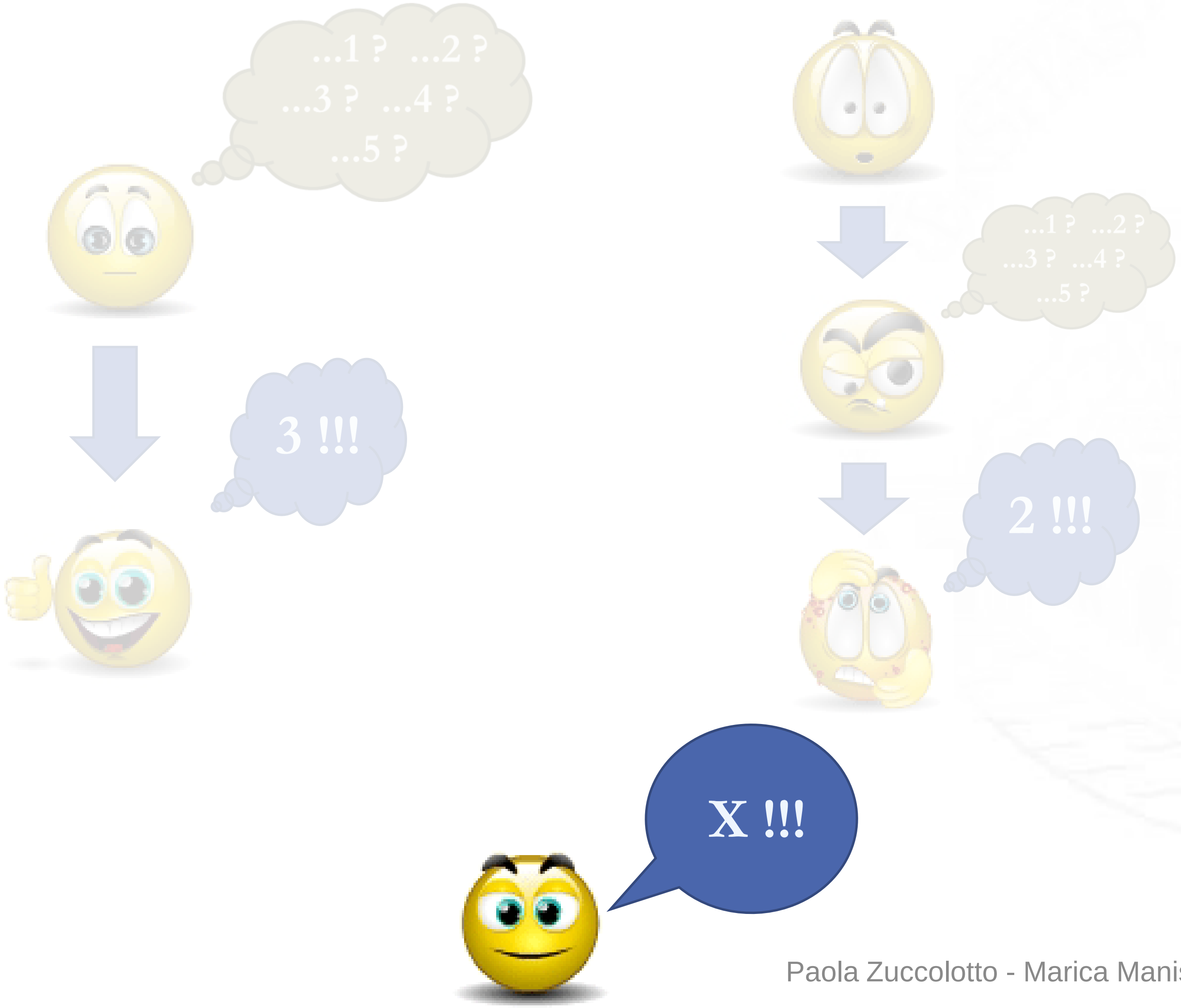
The unconscious Decision Process (DP)

Do you agree with ABC?

Express a rating from 1 (=totally disagree) to 5 (=totally agree)



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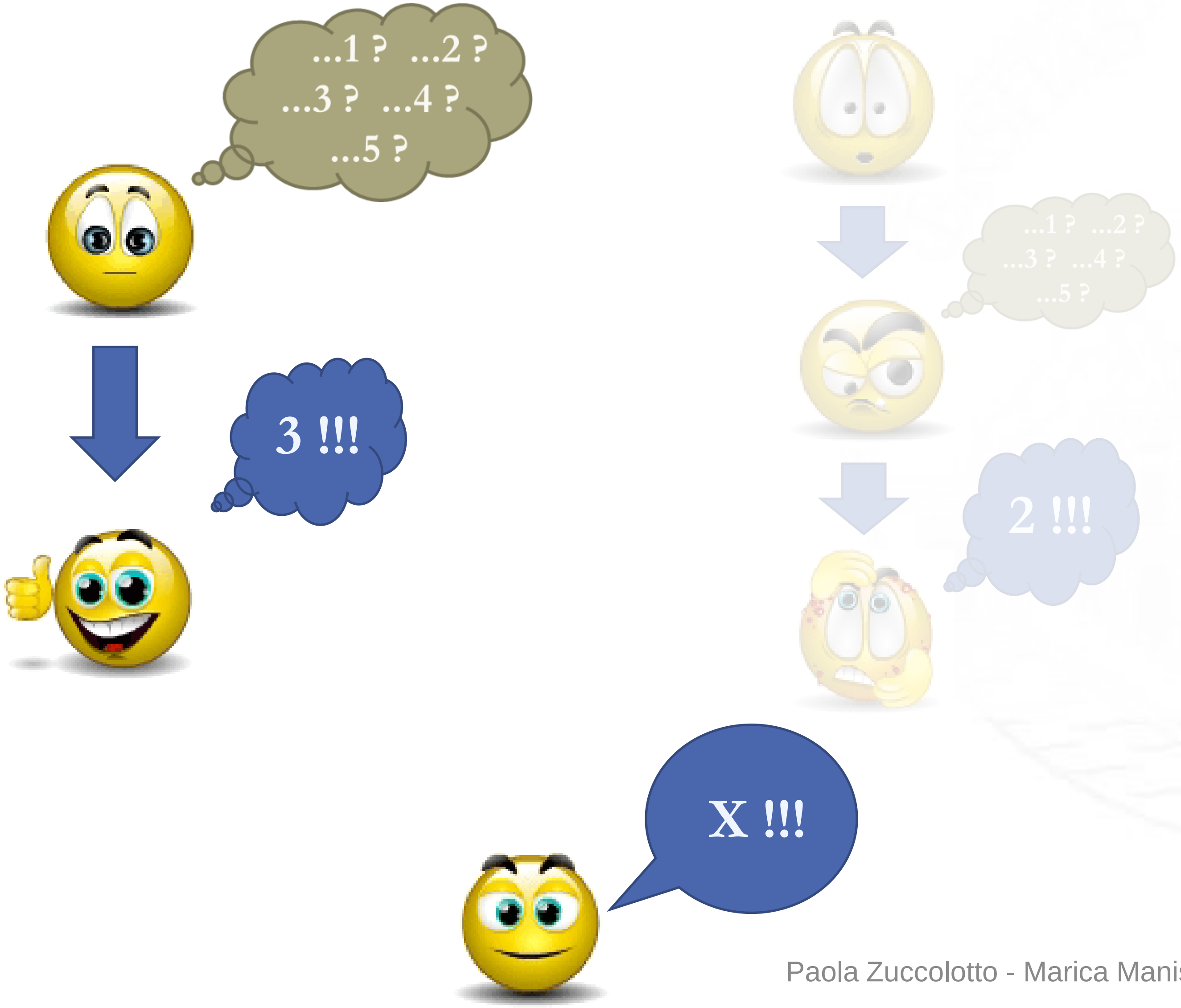

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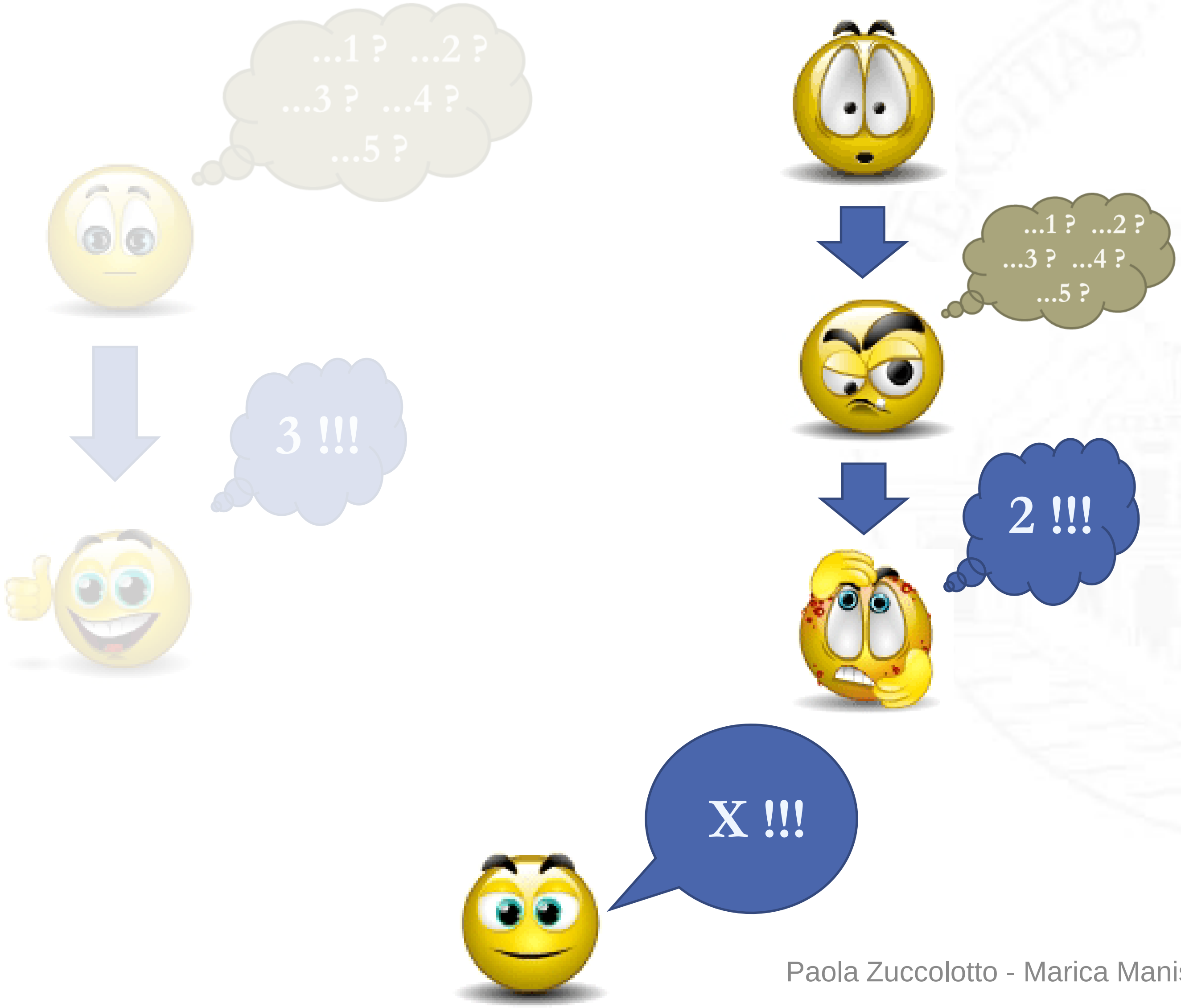

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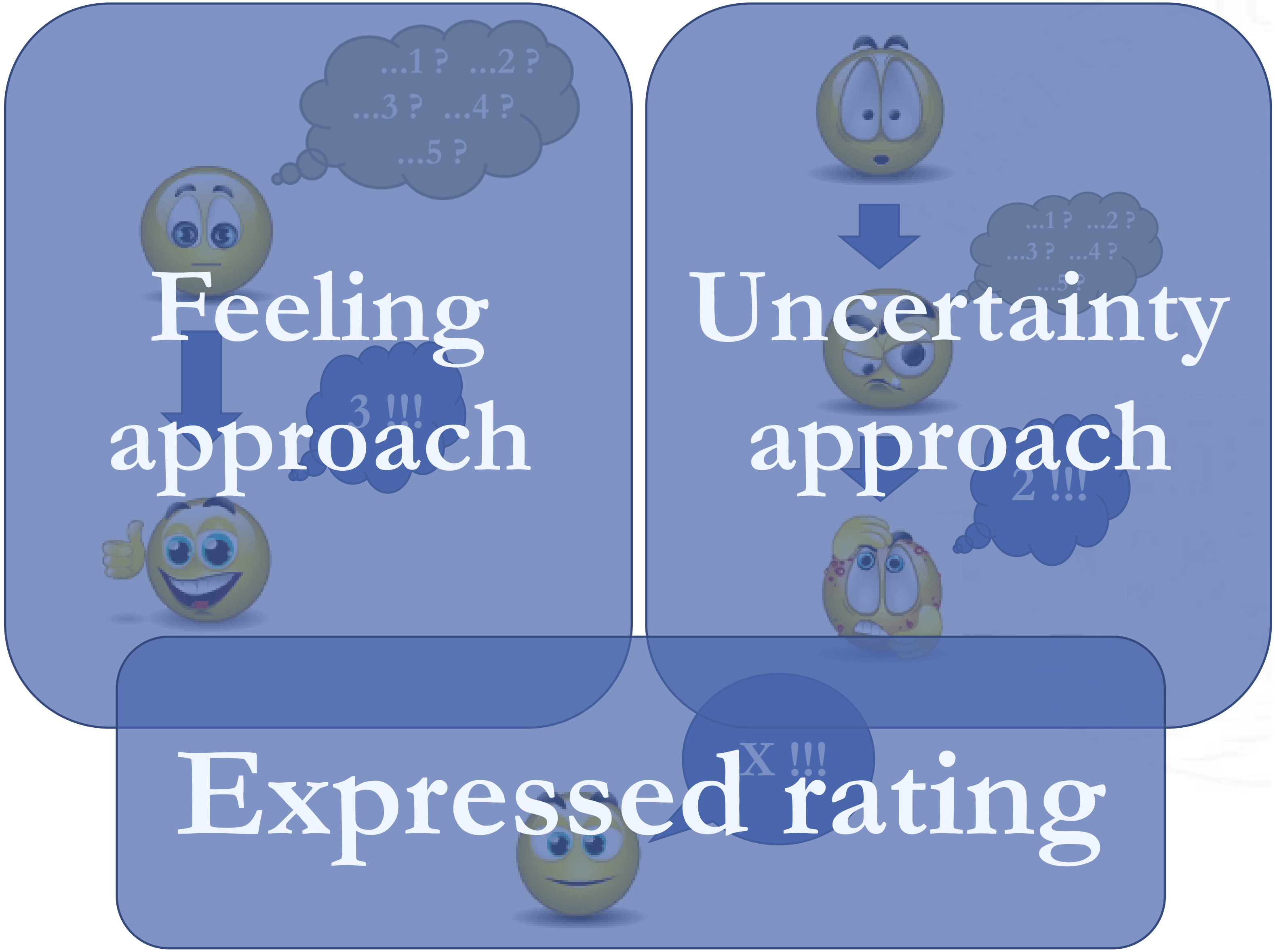

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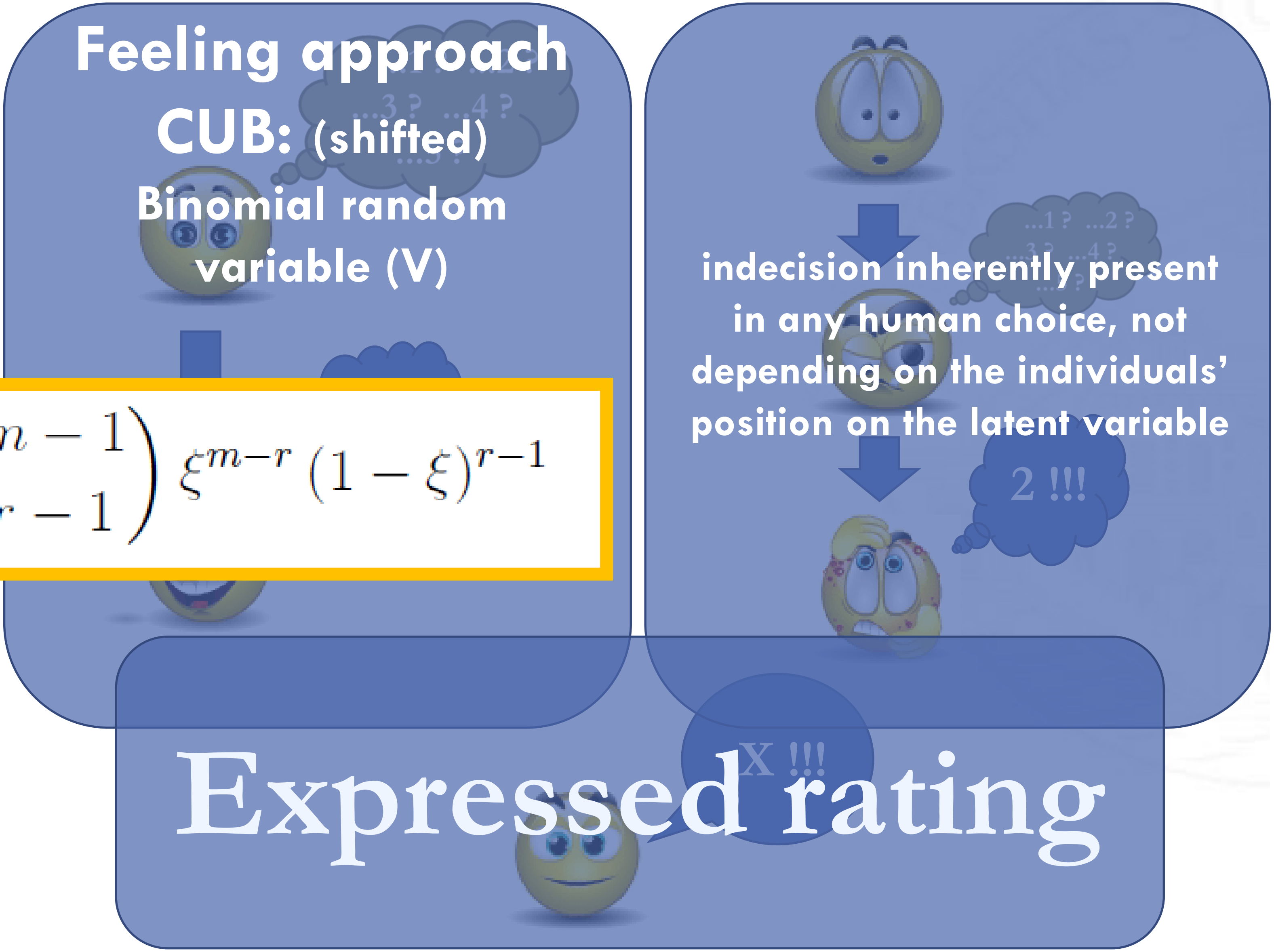

$$b_r(\xi) = P(V = r) = \binom{m-1}{r-1} \xi^{m-r} (1-\xi)^{r-1}$$

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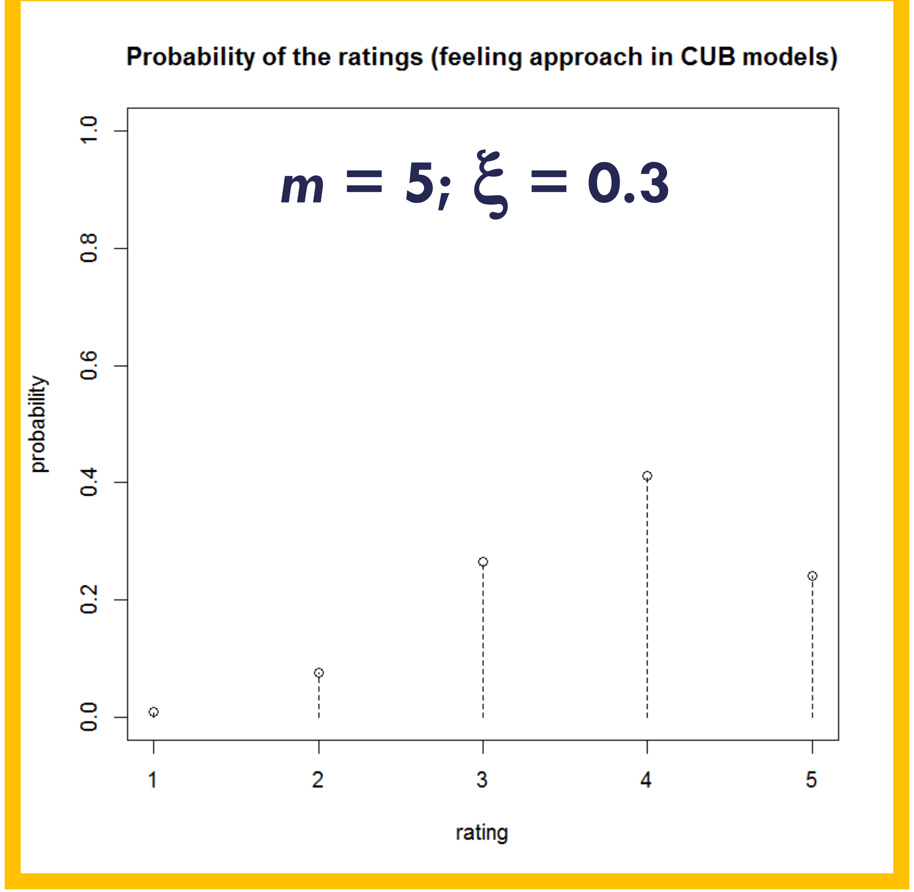
How do CUB models fit into this framework?

The unconscious Decision Process (DP)



Feeling approach

CUB: (shifted) Binomial random variable (V)

$$b_r(\xi) = P(V = r) = \binom{m-1}{r-1} \xi^{m-r} (1-\xi)^{r-1}$$

indecision inherently present in any human choice, not depending on the individuals' position on the latent variable

Expressed rating

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The unconscious Decision Process (DP)

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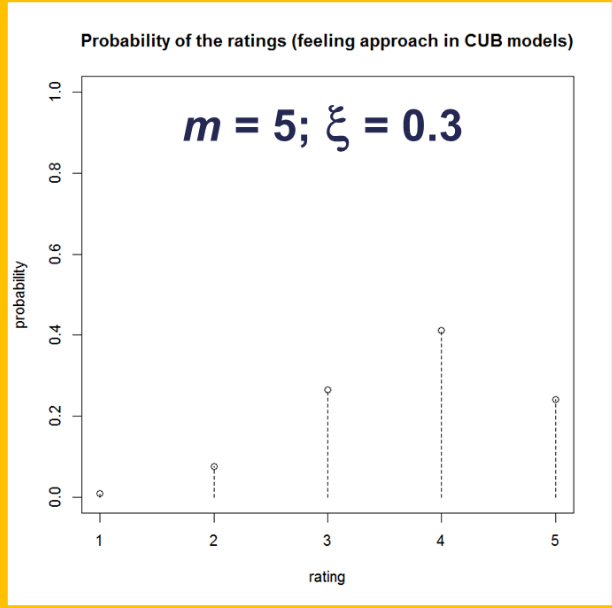

$$b_r(\xi) = P(V = r) = \binom{m-1}{r-1} \xi^{m-r} (1-\xi)^{r-1}$$

Uncertainty approach CUB:
Uniform random variable (U)

$$P(U = r) = 1/m$$

Expressed rating

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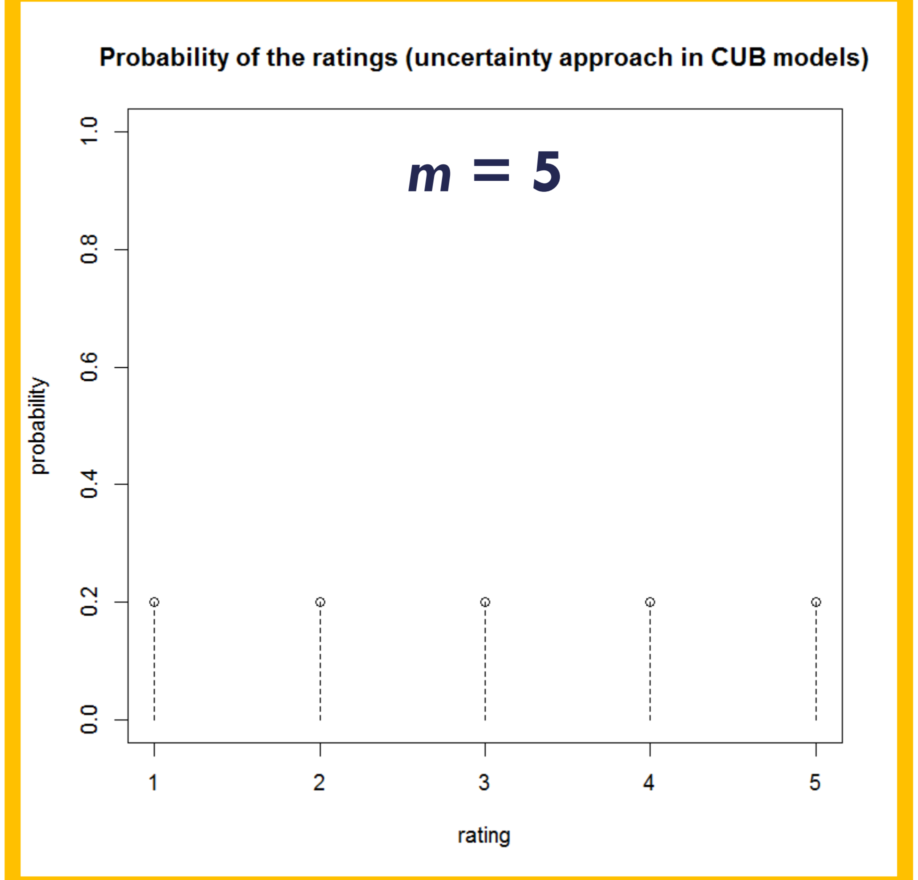

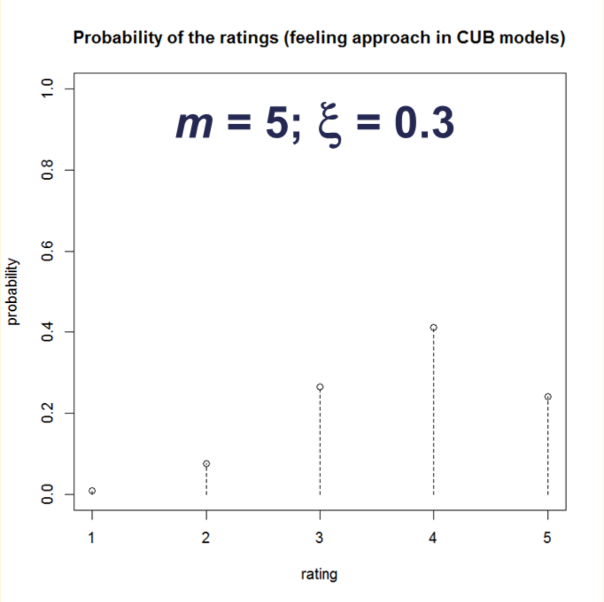

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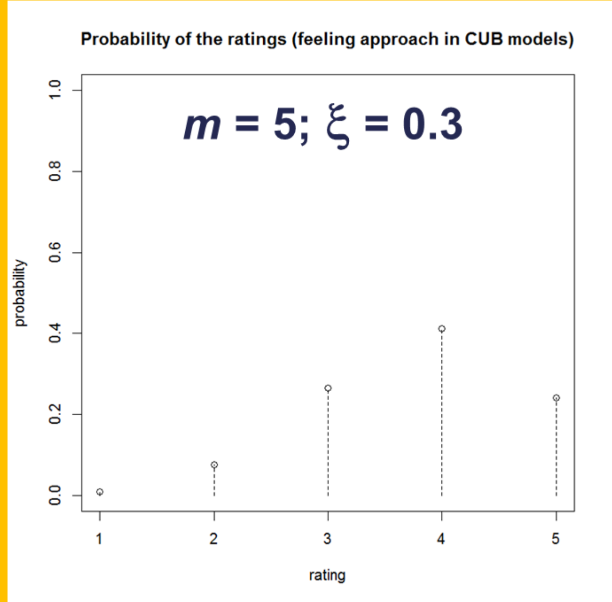

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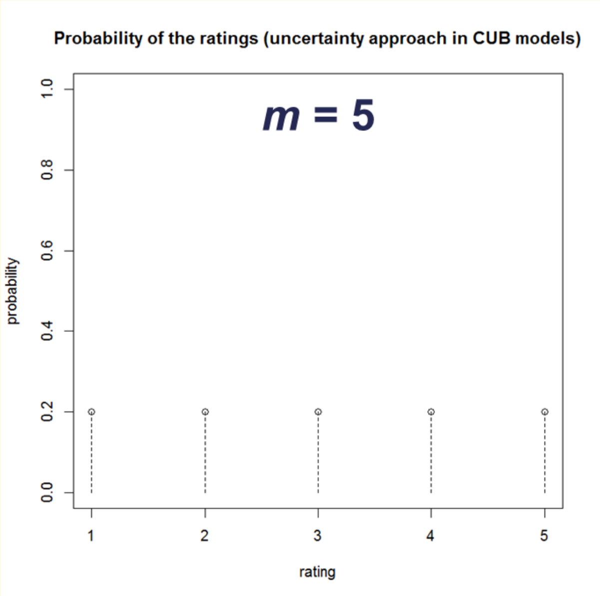

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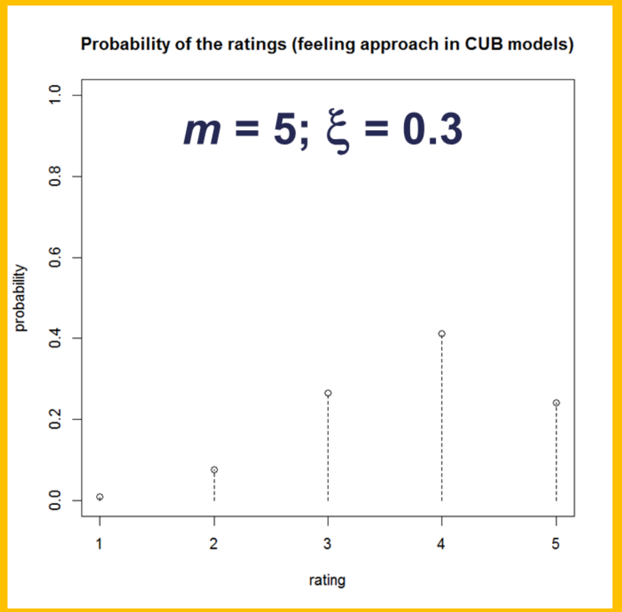

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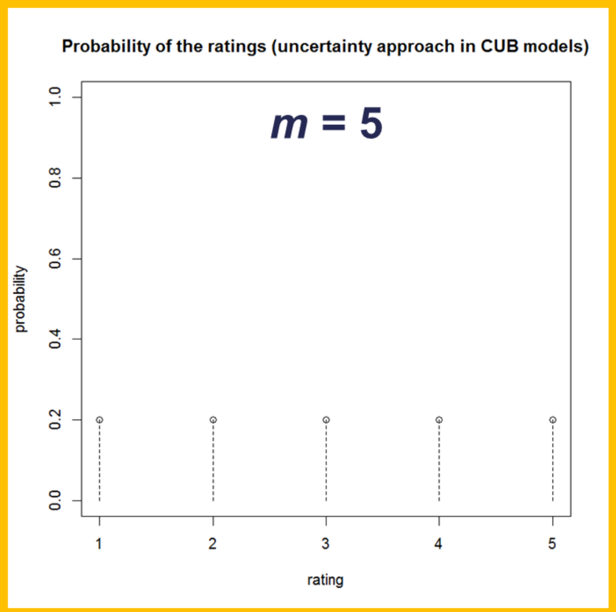

$$P(U = r) = 1/m$$

Expressed rating CUB: mixture of V and U (R)

$$P(R = r \mid \boldsymbol{\theta}) = \pi \, b_r(\xi) + (1-\pi) \, P(U = r)$$

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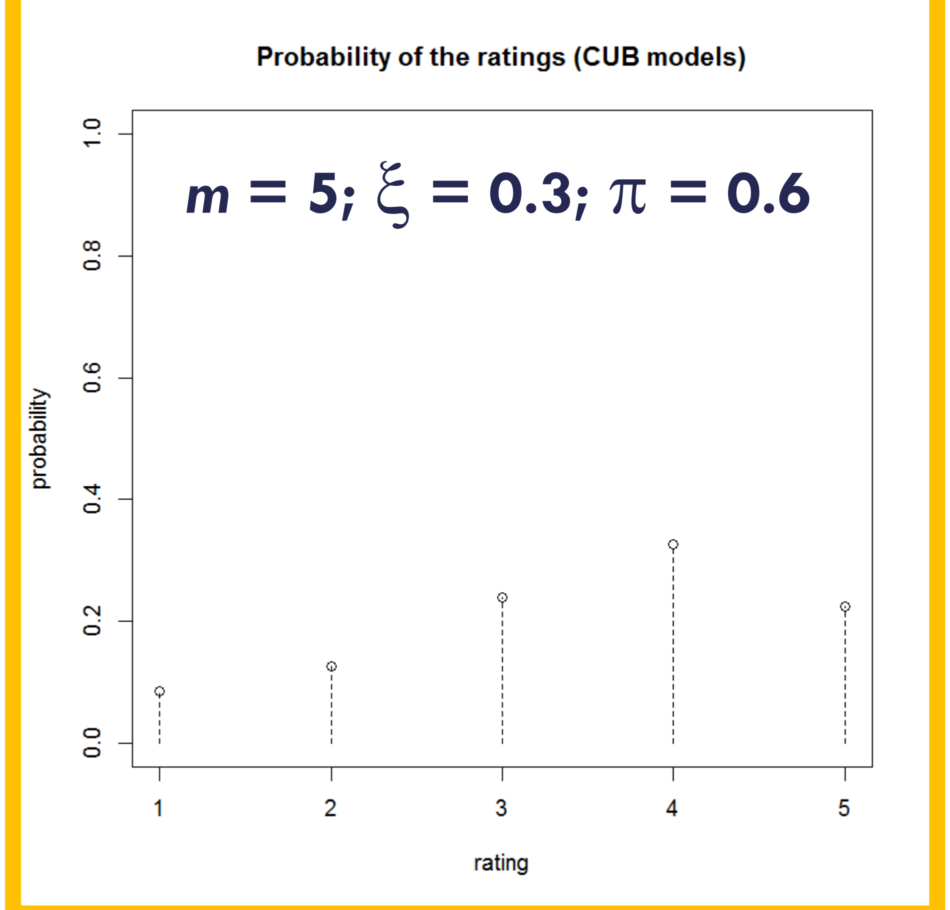
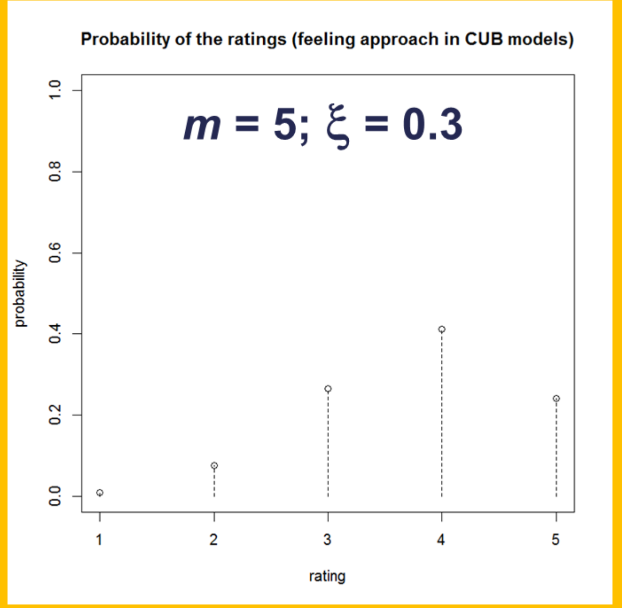
How do CUB models fit into this framework?

The unconscious Decision Process (DP)





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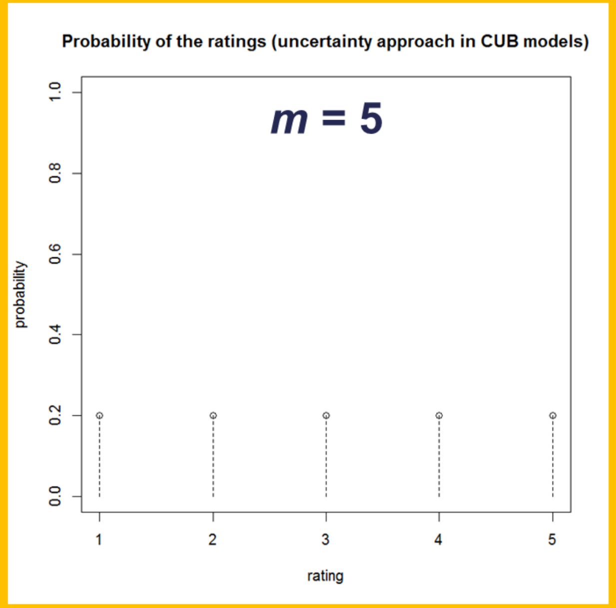

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The unconscious Decision Process (DP)

Do you agree with ABC?
Express a rating from 1 (=totally disagree) to 5 (=totally agree)

Feeling approach CUB: (shifted) **Binomial random variable (V)**

Feeling parameter:
$1 - \xi$

Uncertainty approach CUB: Uniform random variable (U)

Uncertainty parameter:
$1 - \pi$

Expressed rating CUB: mixture of V and U (R)

$$P(R = r \mid \boldsymbol{\theta}) = \pi\, b_r(\xi) + (1 - \pi)\, P(U = r)$$

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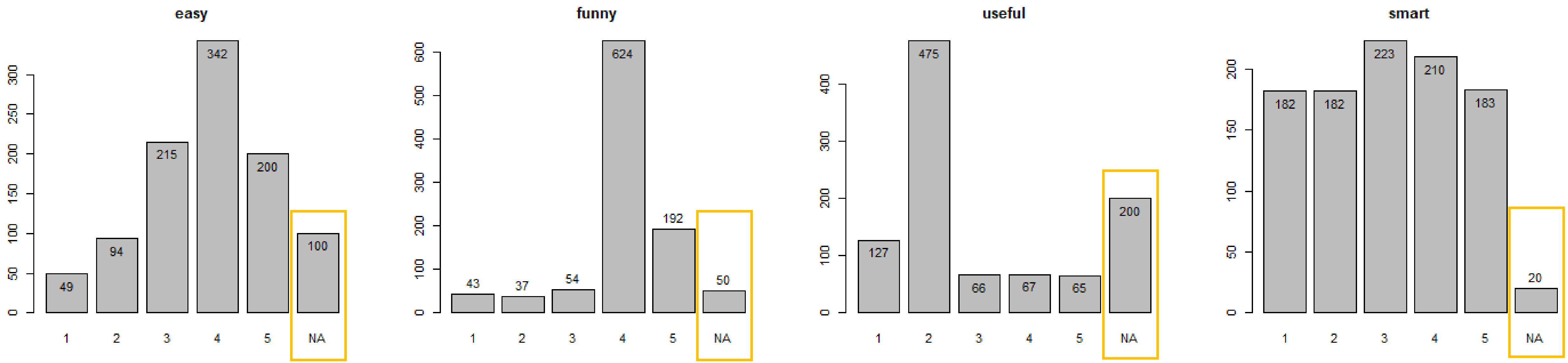
Rating data – example 1 (N=1000, 5-point Likert scale)



With standard CUB model, NAs (DK responses) are removed

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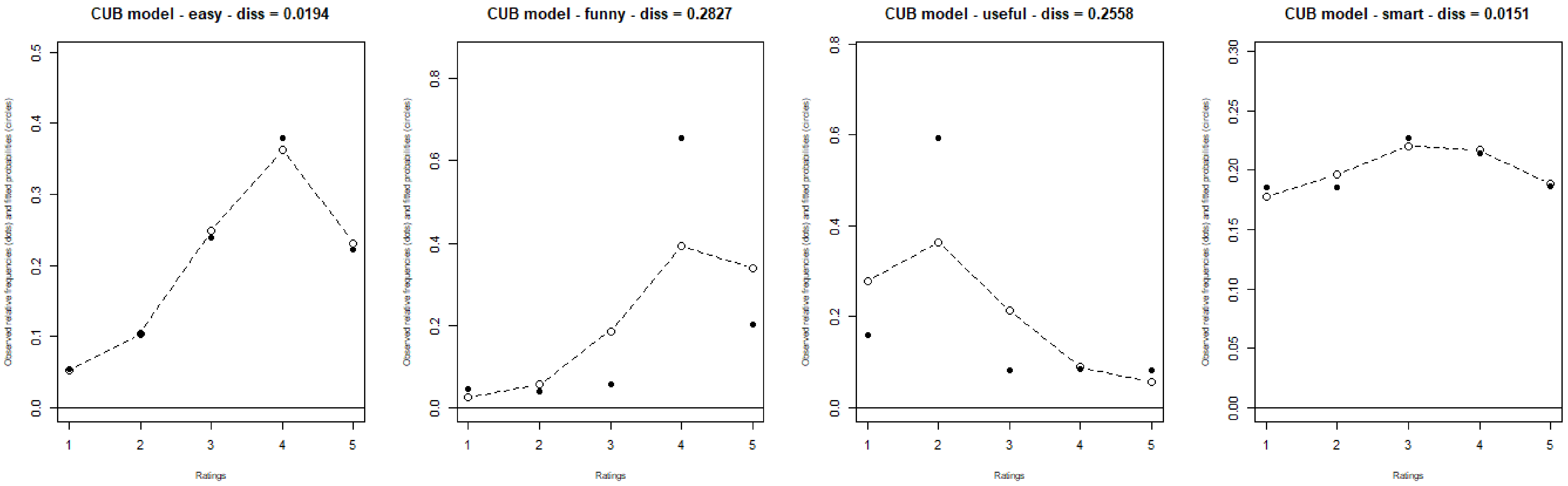

Note: a **CUB model with shelter** could be fit to "funny" and "useful". This would significantly improve results (see Iannario M., 2012, Modelling shelter choices in a class of mixture models for ordinal responses. *Statistical Methods and Applications*, 21:1–22.)

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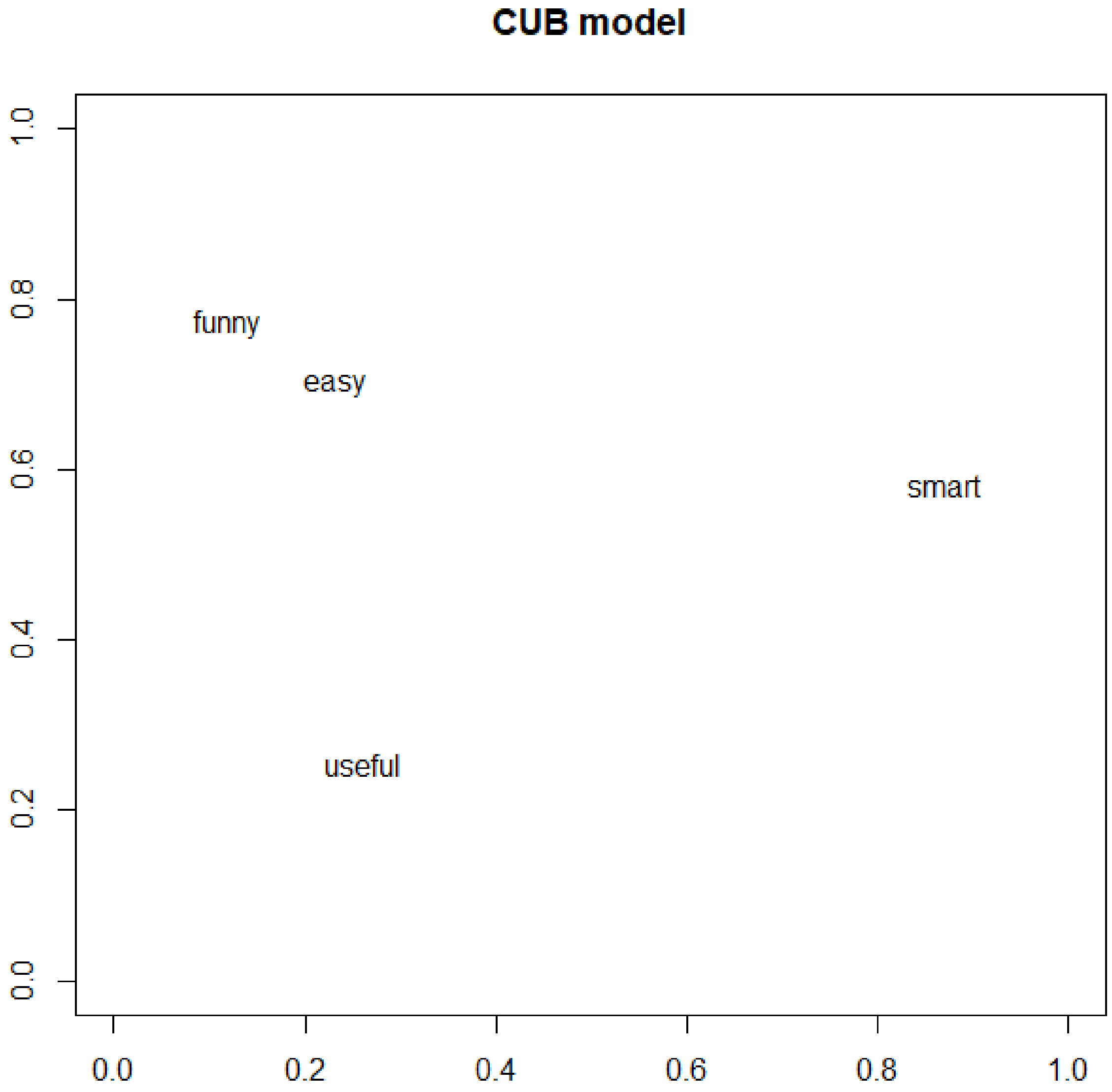
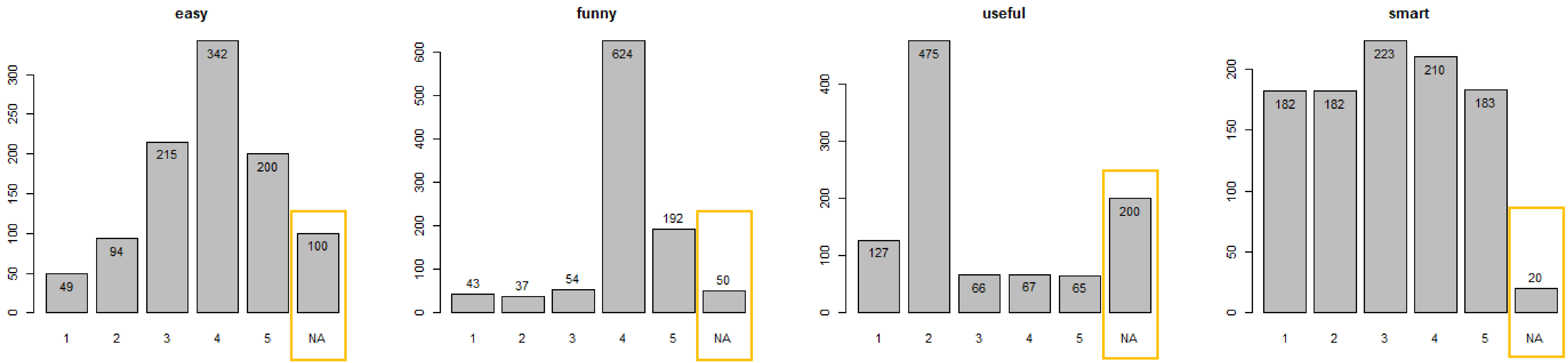


Feeling parameter: $1 - \xi$

Uncertainty parameter: $1 - \pi$

Rating data – example 1 (N=1000, 5-point Likert scale)



Manisera&Zuccolotto, Pattern Recognition Letters

propose to add DK responses to the model, thus treating them as relevant information instead of missing data.

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"Don't know" responses (DK)

Fraction of respondents

Fraction of non respondents (DK)

$$P(R=r|\theta)=f[\pi b_r(\xi)+(1-\pi)P(U=r)]+(1-f)P(U=r)$$

CUB model for respondents

Probability distribution of rating assumed for DK responses

$$P(R=r|\theta)=\pi_{adj}b_r(\xi)+\left(1-\pi_{adj}\right)P(U=r)$$

with $\pi_{adj}=f\pi$

Considering DK responses as relevant information instead of missing information leads to a CUB model with adjusted uncertainty parameter

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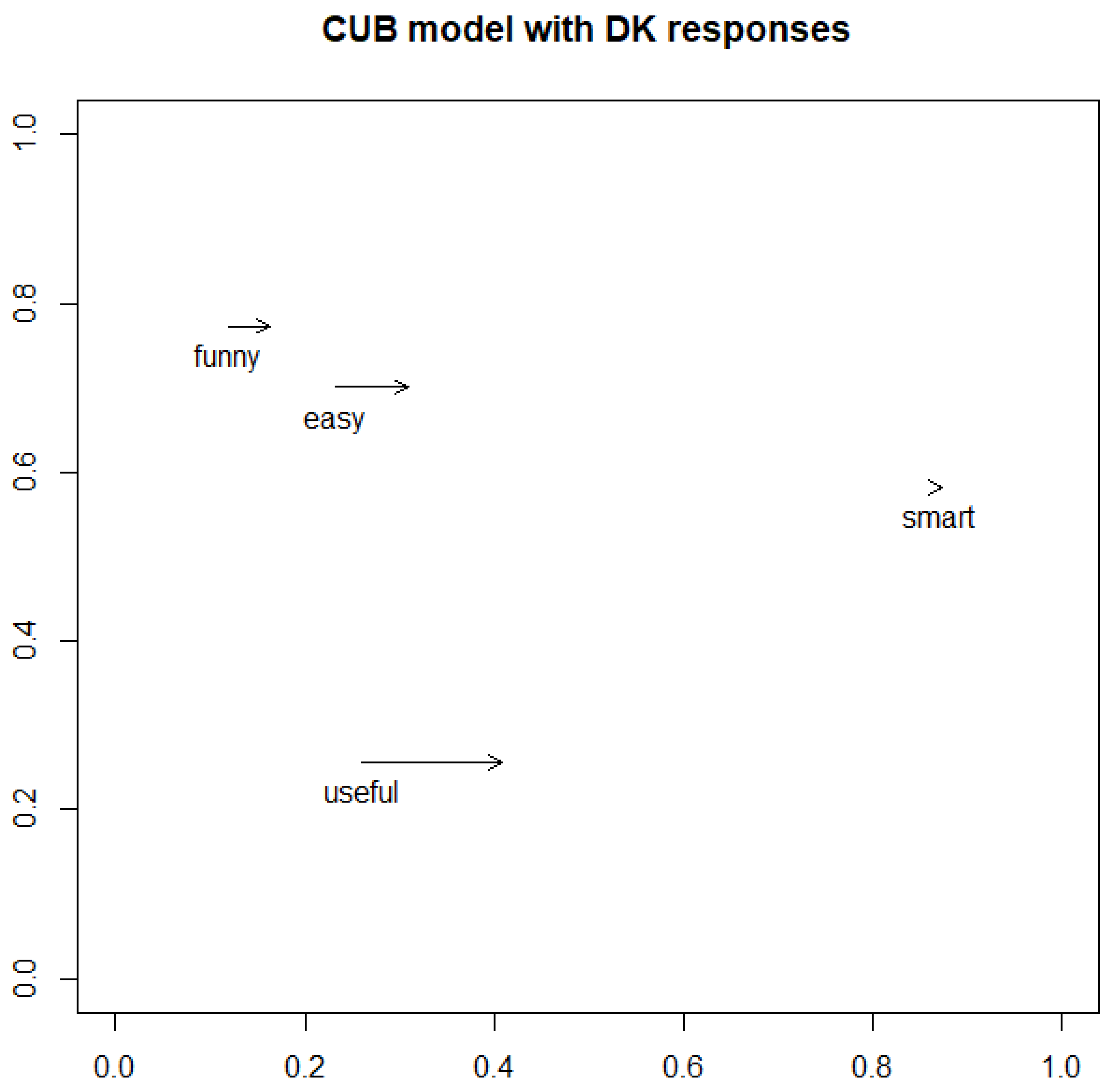

Feeling parameter:
$1 - \xi$

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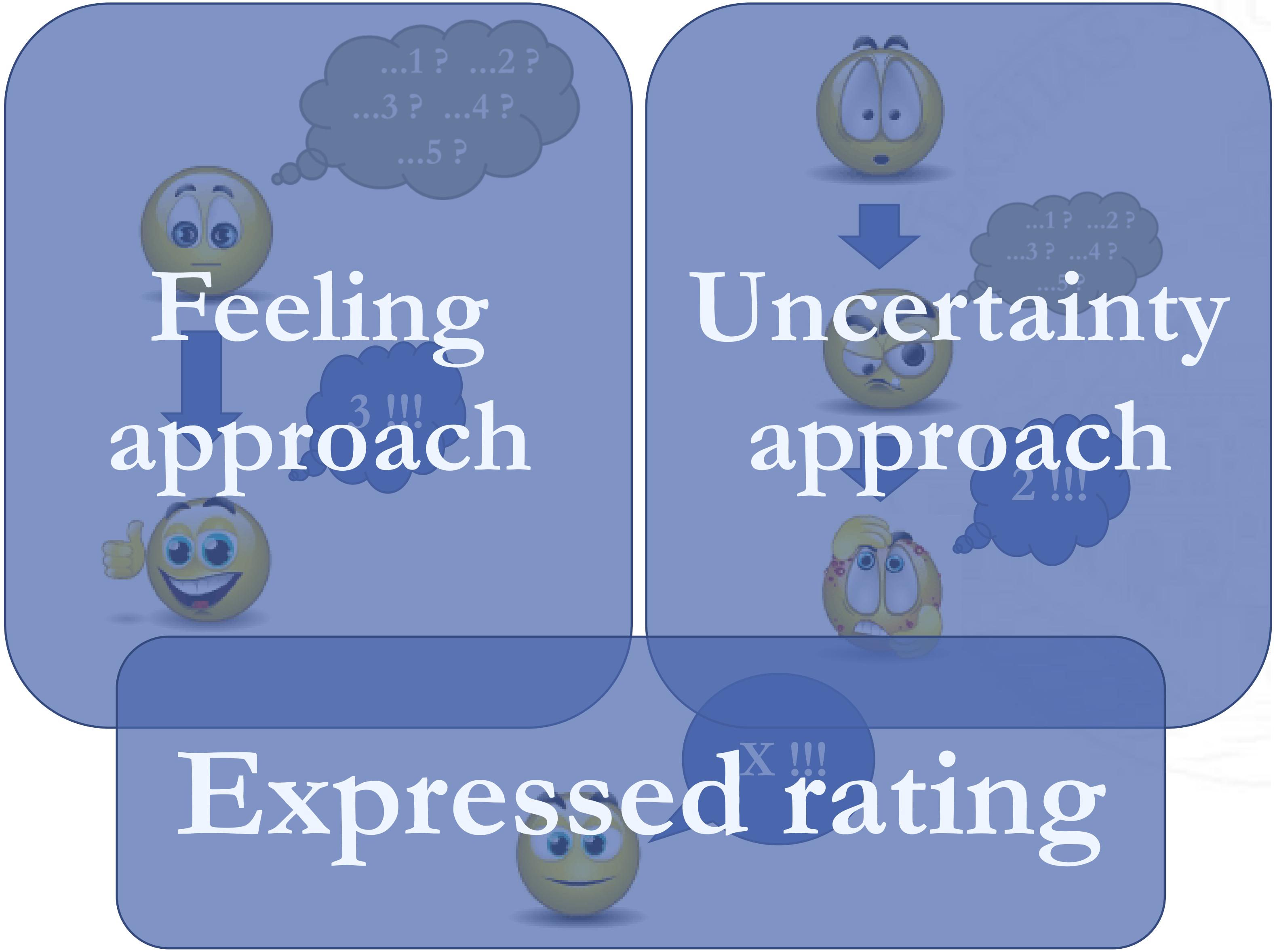
The unconscious Decision Process (DP)

Focus on the **Feeling approach**



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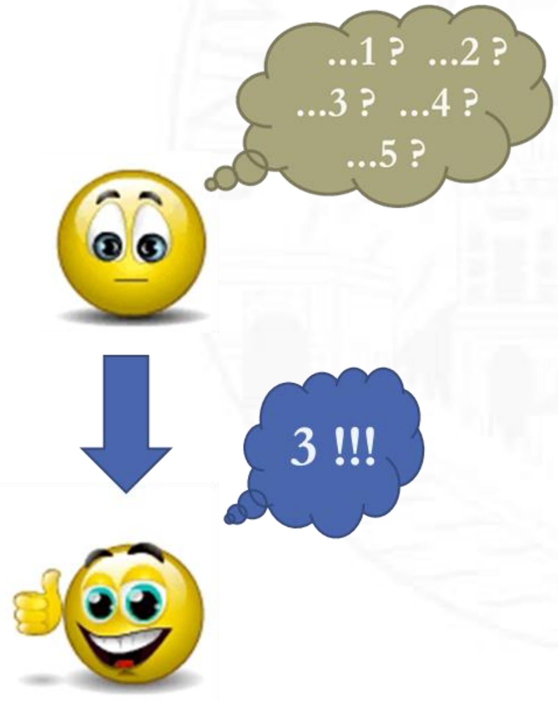
The unconscious Decision Process (DP)

- We assume that the Feeling approach proceeds through *T* consecutive **steps.**
- At each step a **basic judgment** is formulated.
- Step-by-step, the basic judgments are **accumulated** and **transformed** into provisional ratings.
- The rating at the end of the Feeling approach is given by the **last provisional rating.**

Do you agree with ABC? Express a rating from 1 (=totally disagree) to 5 (=totally agree)



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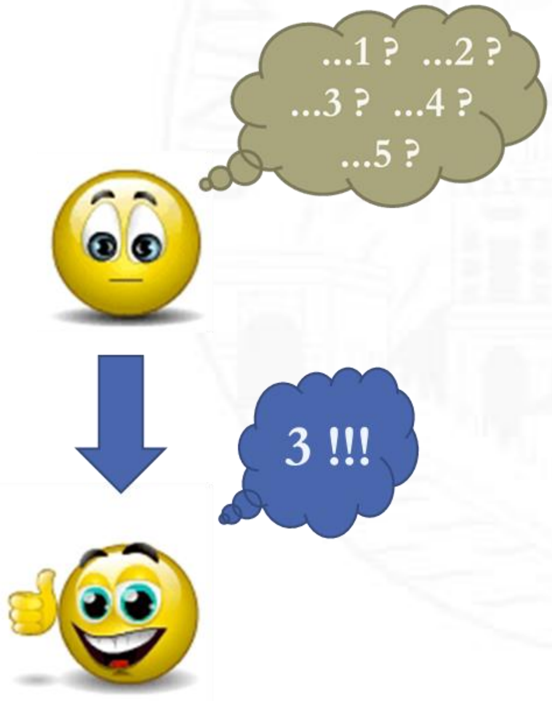

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An assessment about the latent trait, but a simpler task than the full rating expression. Example: gather thoughts around a single aspect of the statement under evaluation and decide whether you agree or not (Yes/No)

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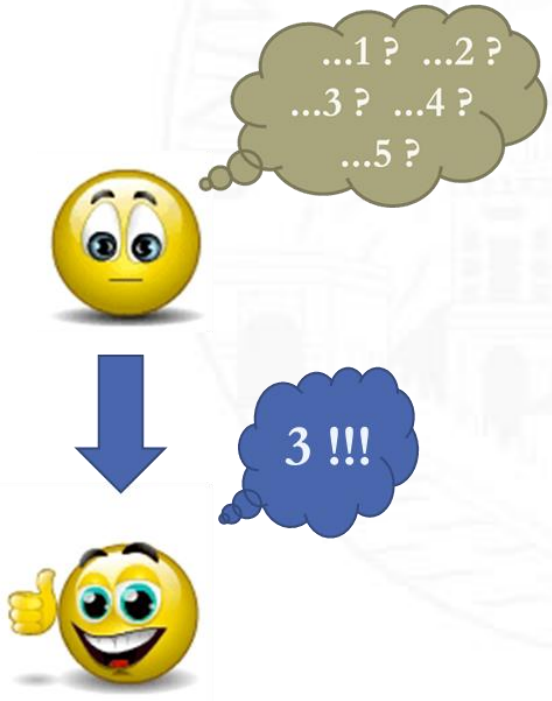

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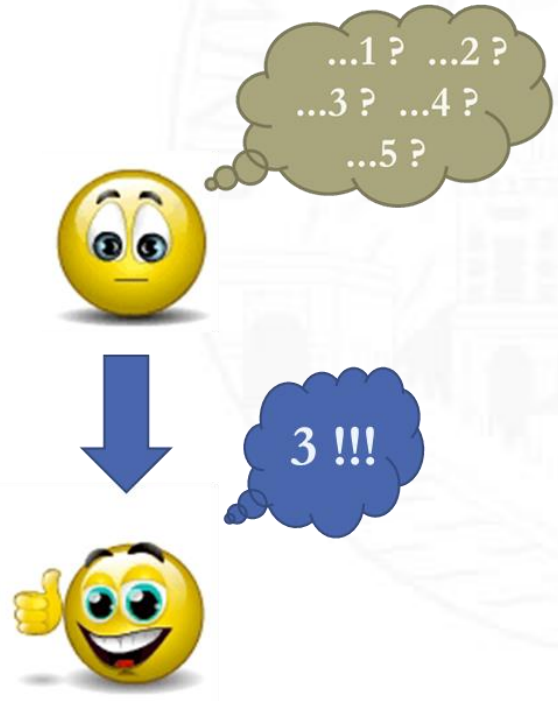

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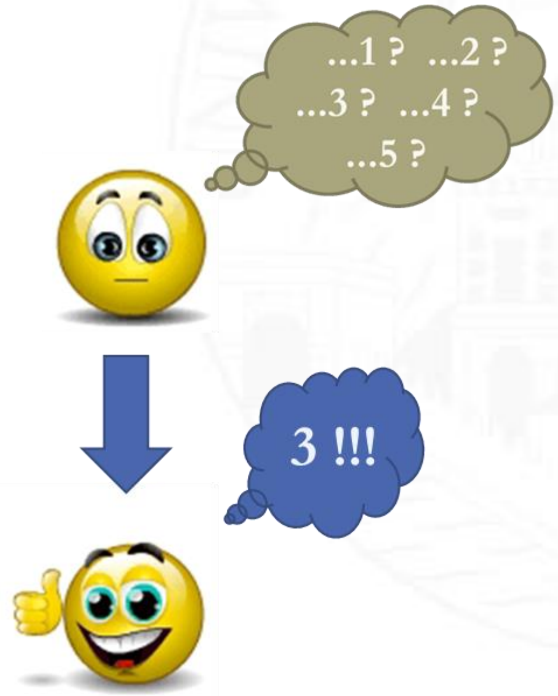

The unconscious Decision Process (DP)

A) Feeling Approach

1. *Elementary judgments:* An *iid* sequence of random variables $X_1, \cdots, X_T$ with domains $\mathcal{D}_{X_1}, \cdots, \mathcal{D}_{X_T}$ generates T elementary judgments $x_1, \cdots, x_T$ progressively expressed along T steps.
2. *Accumulating function:* At each step t, a function $f : \mathcal{D}_{X_1} \times \cdots \times \mathcal{D}_{X_t} \to \Psi_t \subseteq \mathbb{R}$ summarizes the t past elementary judgments (for example, by summation). We say that f is an accumulating function, i.e. we require it obeys the following property: $\Psi_t \subseteq \Psi_{t+1}, \forall t$.
3. *Accumulated judgments:* A sequence of random variables $W_1, \cdots, W_T$, $W_t = f(X_1, \cdots, X_t)$, with domains $\mathcal{D}_{W_1} \equiv \Psi_1, \cdots, \mathcal{D}_{W_T} \equiv \Psi_T$ is then originated along the T steps of the DP with T corresponding realizations $w_1, \cdots, w_T$, $w_t = f(x_1, \cdots, x_t)$, called accumulated judgments.
4. *'Likertization' function:* At each step t, a non-decreasing function $d : \mathcal{D}_{W_T} \to (1, \cdots, m)$ transforms w_t into a provisional rating. Note that from the definition of accumulating function derives $\mathcal{D}_{W_1} \subseteq \cdots \subseteq \mathcal{D}_{W_T}$, so that d can always be computed on the domain of W_t, for all t.
5. *Provisional ratings:* A sequence of random variables $R_1, \cdots, R_T$, $R_t = d(W_t)$, with domains the space $(1, \cdots, m)$ is then originated along the T steps of the feeling path with T corresponding realizations $r_1, \cdots, r_T$, $r_t = d(w_t)$, called provisional ratings.

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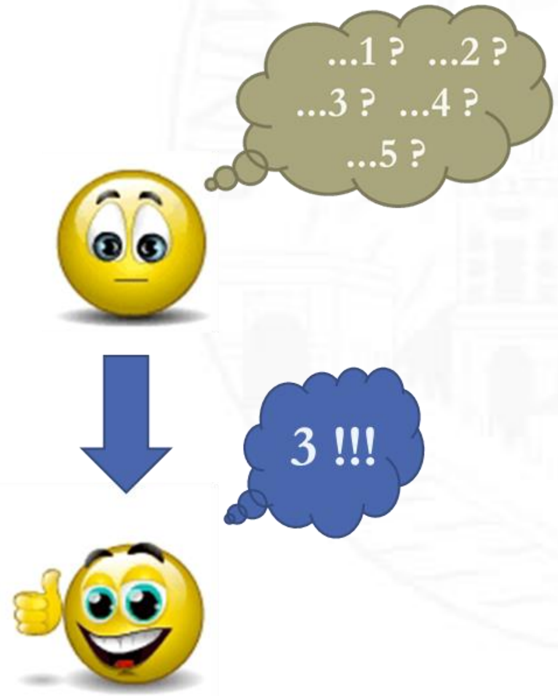

The unconscious Decision Process (DP)

We can obtain several different models, depending on the assumptions we make about:

- the **distribution of the basic judgments**
- the **accumulation function**
- the **transformation function**

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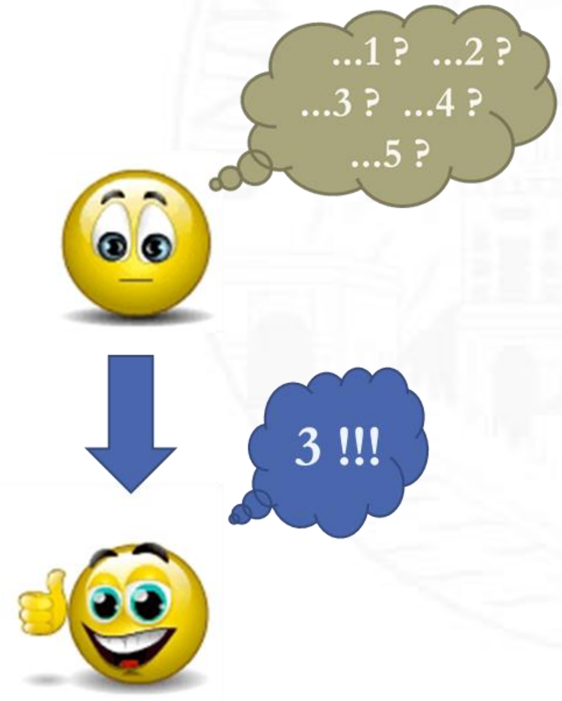
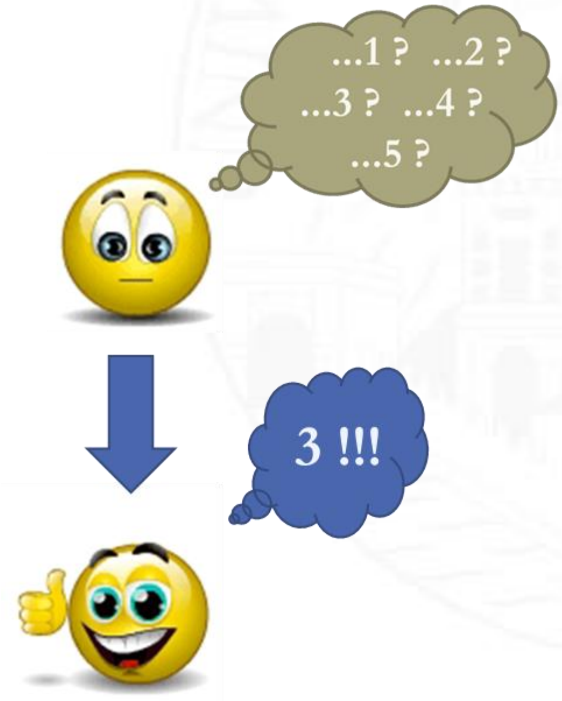


The unconscious Decision Process (DP)

CUB model

- <u>the **distribution of the basic judgments**</u>: m - 1 **Bernoulli** random variables (agree=1 / disagree=0) with success parameter $1 - \xi$. The feeling parameter is then the probability of a positive basic judgment.
- <u>the **accumulation function**</u>: **sum** (which generates a **Binomial** random variable). At each step the number of positive basic judgments is considered.
- <u>the **transformation function**</u>: the accumulated basic judgments + 1 (which generates a **Shifted Binomial** random variable).

Do you agree with ABC? Express a rating from 1 (=totally disagree) to 5 (=totally agree)



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The unconscious Decision Process (DP)

Two new models in the CUB class have been developed by acting on the assumptions of the Feeling approach in the generalized formulation of the DP: the **NLCUB** and the **CUM** model

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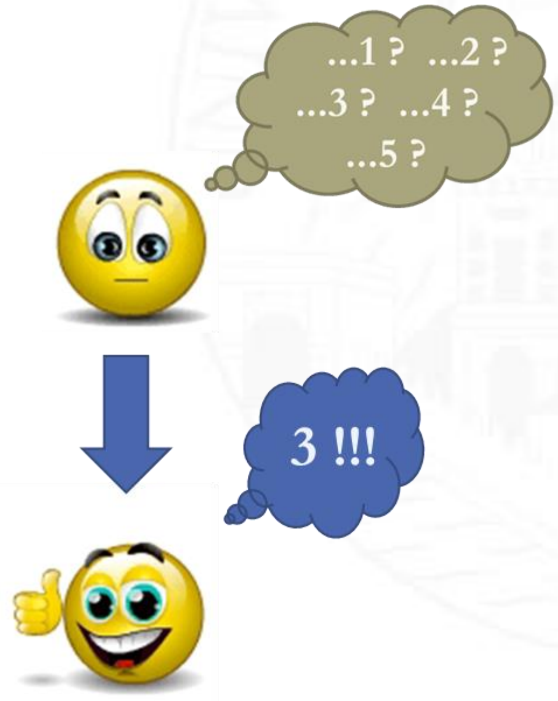

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The unconscious Decision Process (DP)

NLCUB (Non Linear CUB) model

- <u>the **distribution of the basic judgments**</u>: T (> m - 1) **Bernoulli** random variables (agree=1 / disagree=0) with success parameter 1 – ξ. The feeling parameter is then the probability of a positive basic judgment.
- <u>the **accumulation function**</u>: **sum** (which generates a **Binomial** random variable). At each step the number of positive basic judgments is considered.
- <u>the **transformation function**</u>: a **flexible mapping (to be estimated based on data**) onto the Likert scale 1, 2, …, m.

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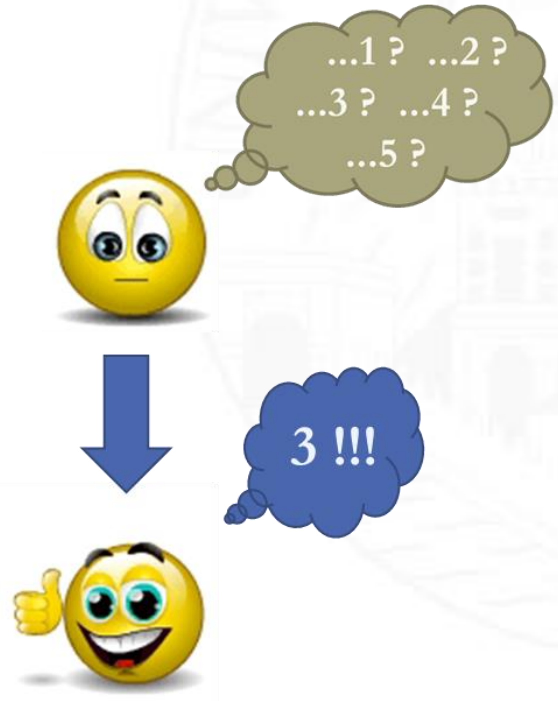
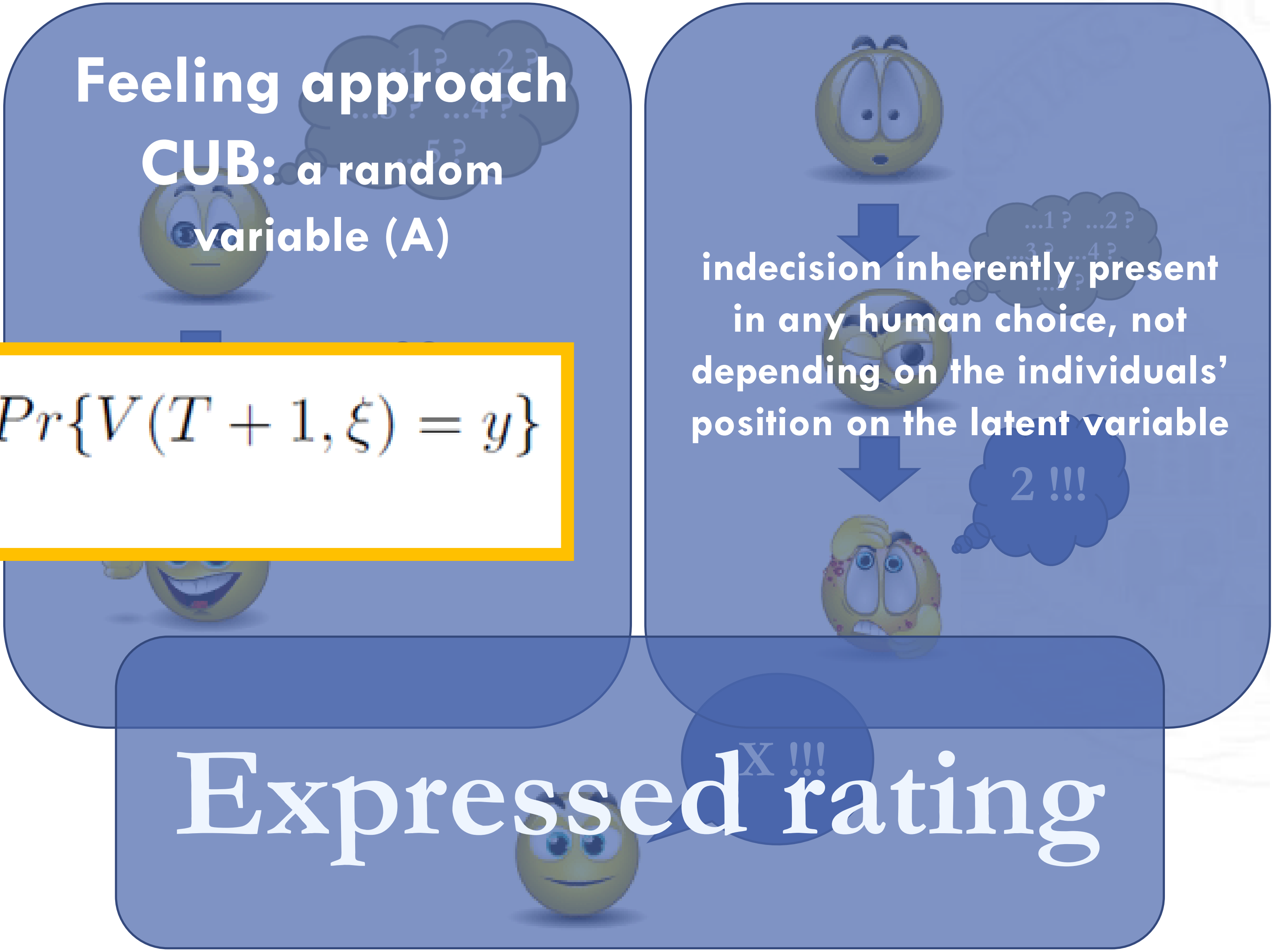


The unconscious Decision Process (DP)

Do you agree with ABC?

Express a rating from 1 (=totally disagree) to 5 (=totally agree)

$$P(A=r) = \sum_{y \in l^{-1}(r)} Pr\{V(T+1,\xi)=y\}$$



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How does NLCUB model fit into this framework?

The unconscious Decision Process (DP)

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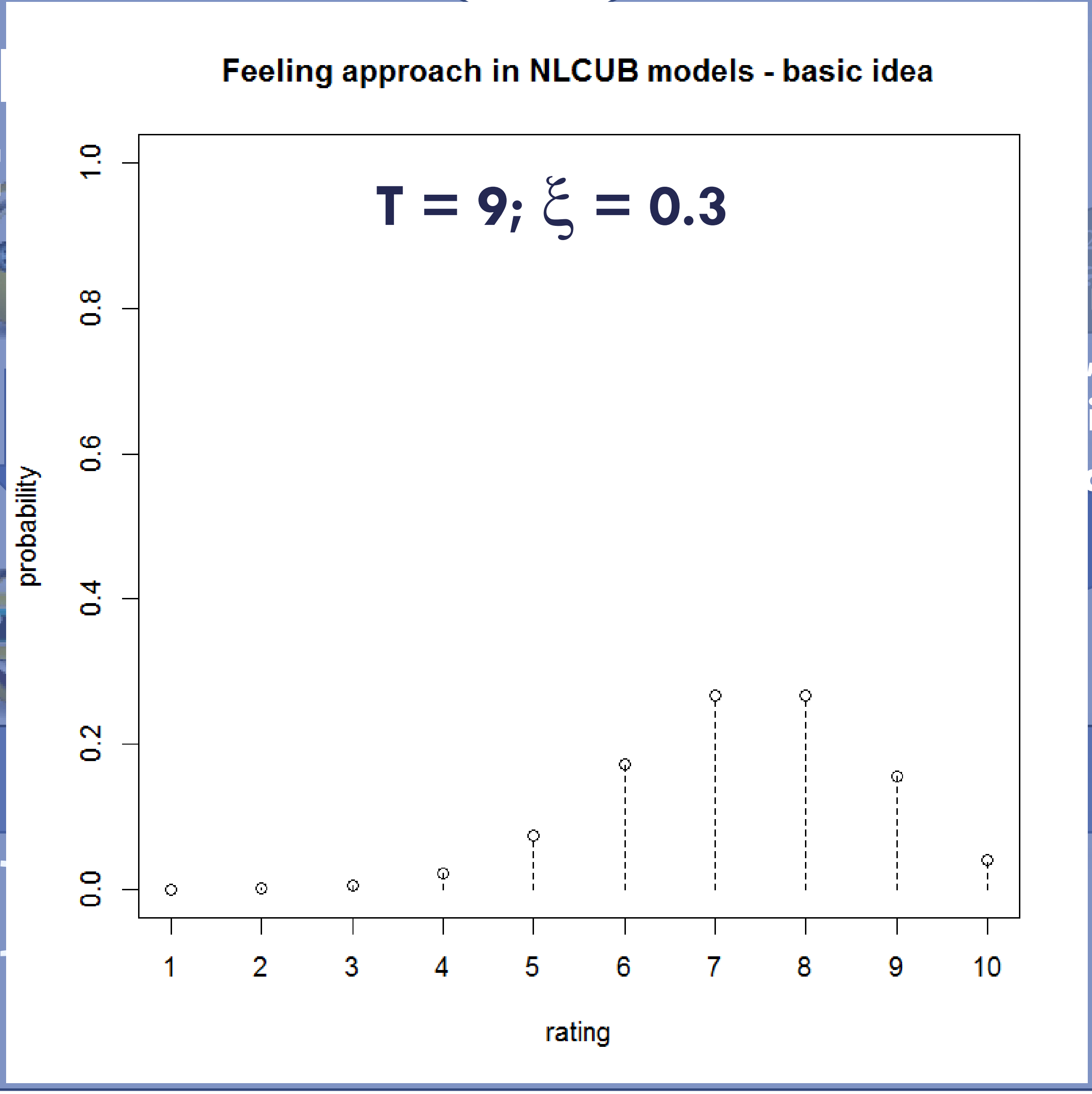

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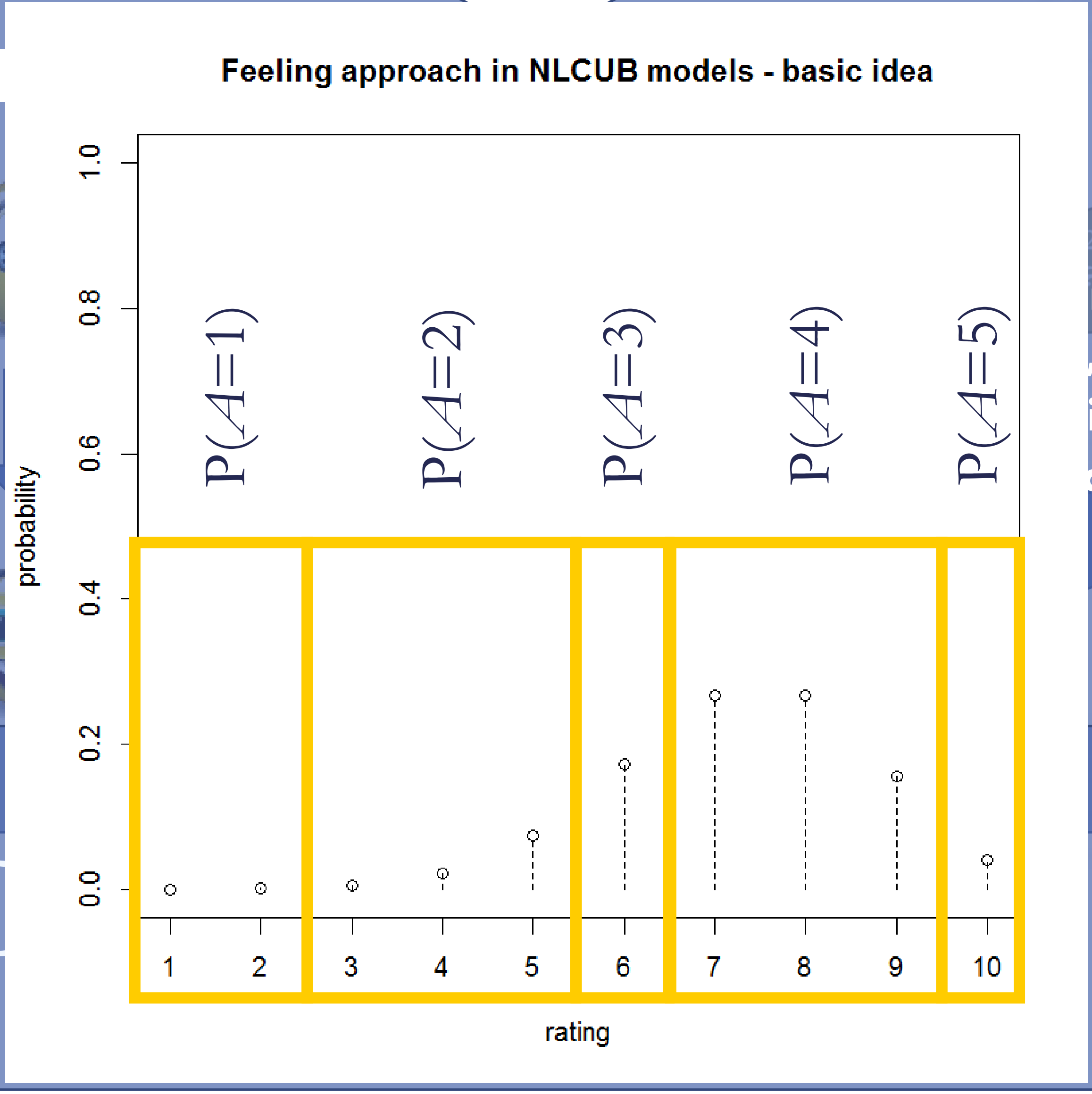
$$P(A=r) = \sum_{y \in l^{-1}(r)} Pr\{V(T+1,\xi)=y\}$$



How does NLCUB model fit into this framework?

The unconscious Decision Process (DP)

Do you agree with ABC?

Express a rating from 1 (=totally disagree) to 5 (=totally agree)

$$P(A=r) = \sum_{y \in l^{-1}(r)} Pr\{V(T+1,\xi) = y\}$$

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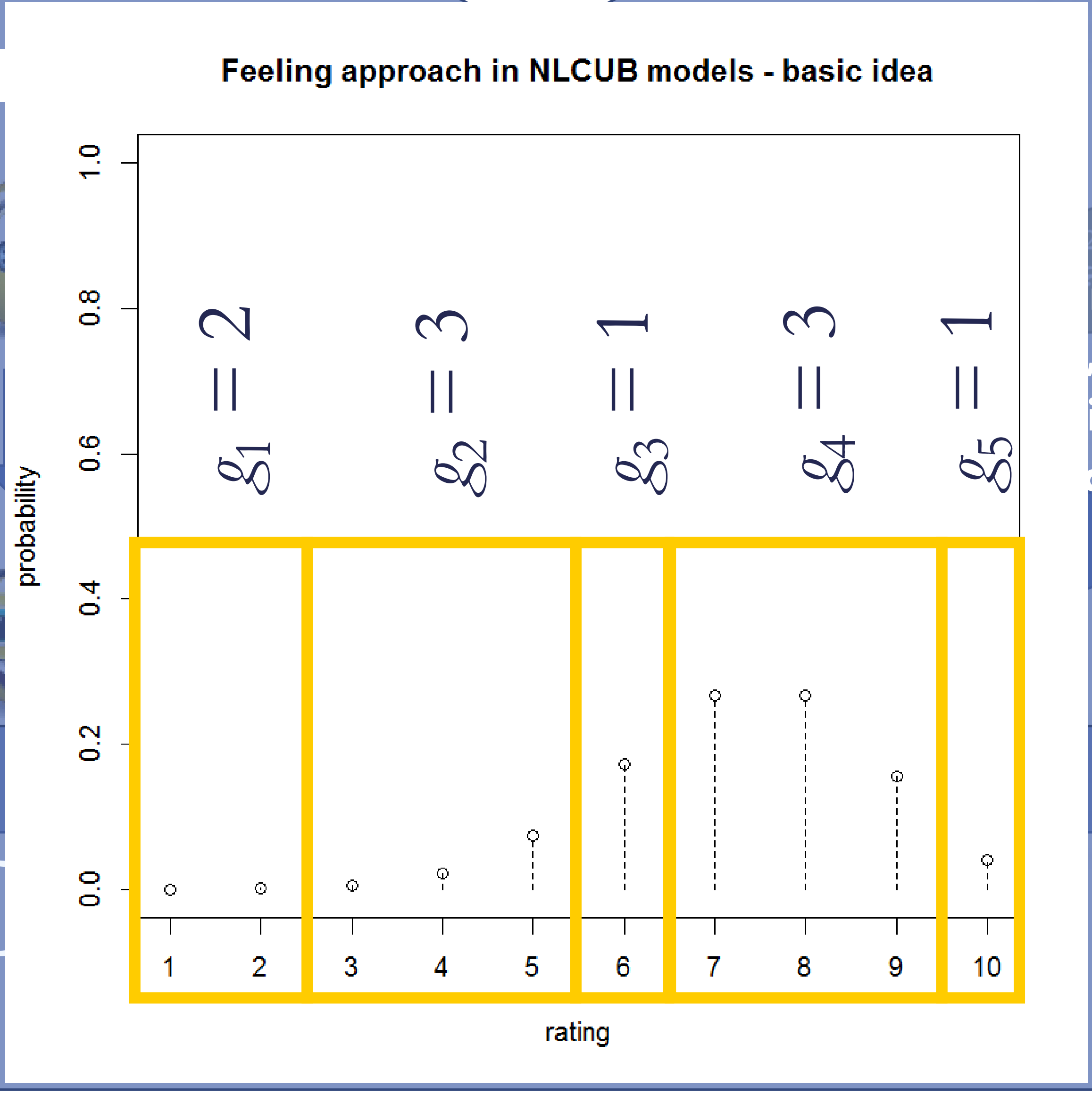

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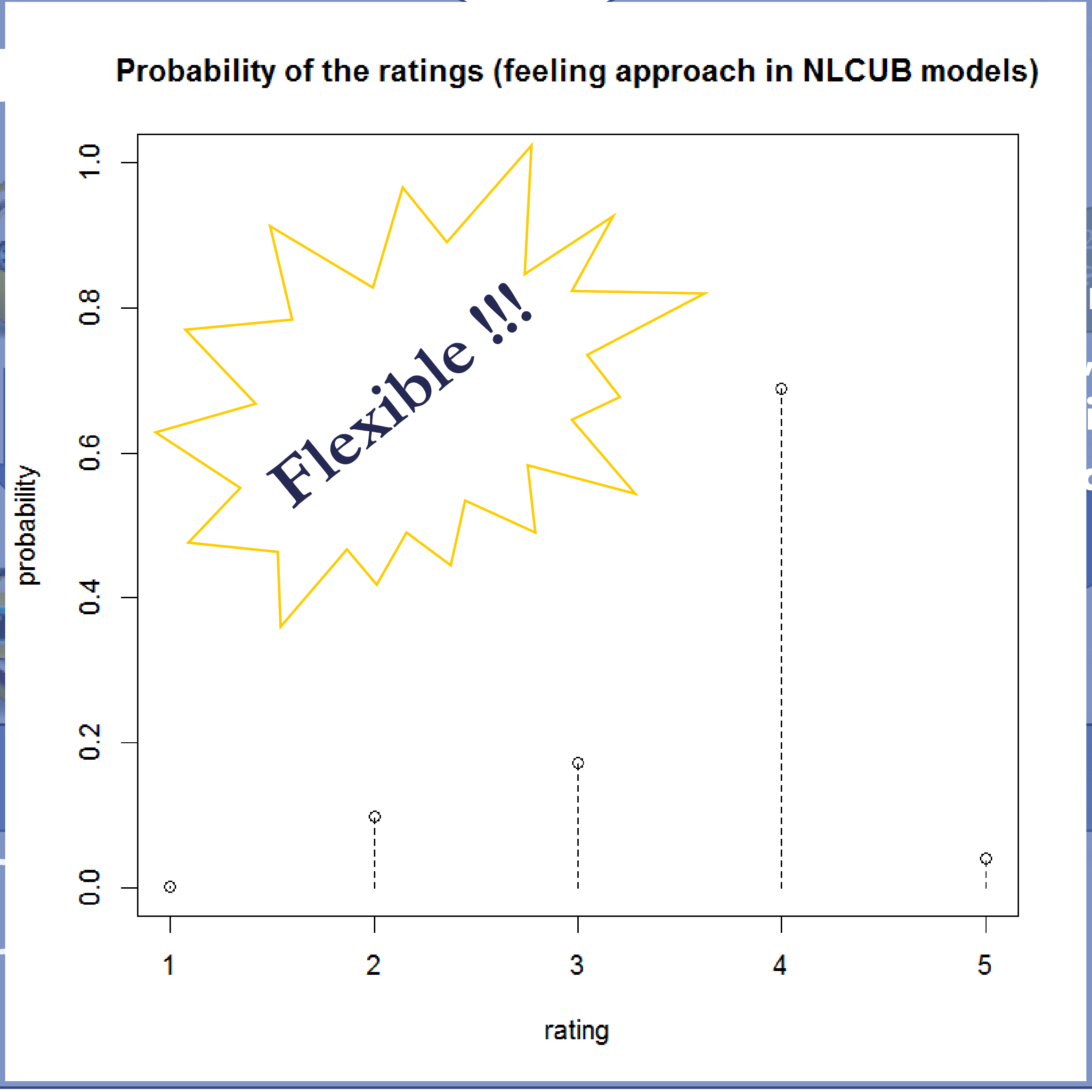

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How does NLCUB model fit into this framework?

The unconscious Decision Process (DP)

Do you agree with ABC?

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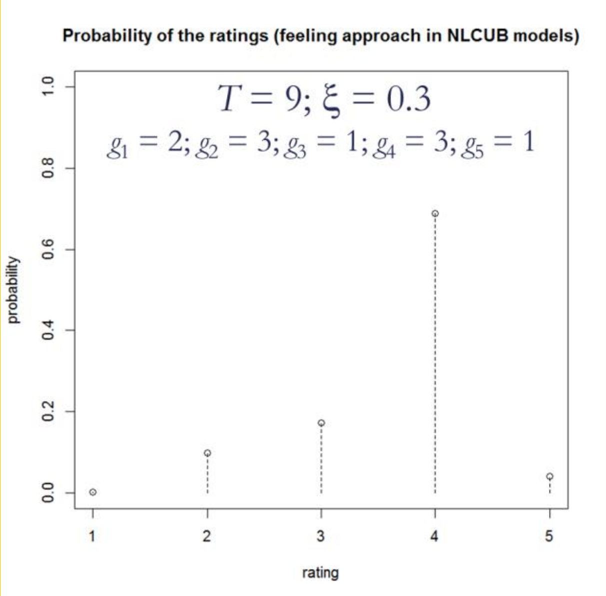
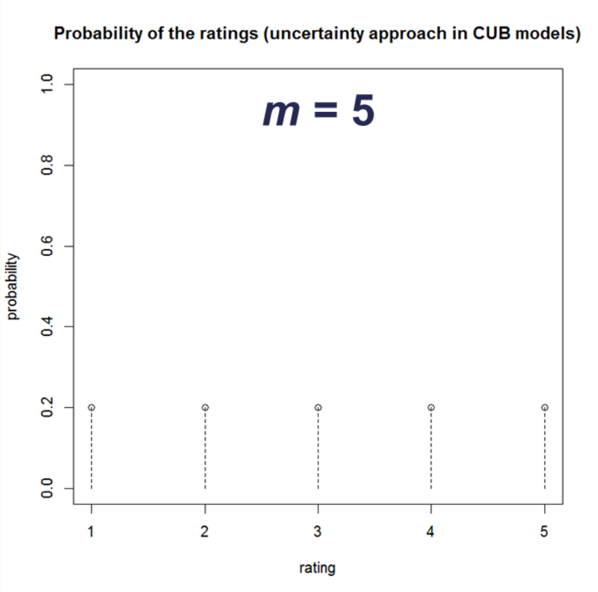


$$P(A = r) = \sum_{y \in l^{-1}(r)} Pr\{V(T+1, \xi) = y\}$$



$$P(U = r) = 1/m$$

Expressed rating NLCUB: mixture of A and U (R)

$$P(R = r \mid \boldsymbol{\theta}) = \pi\, P(A = r) + (1 - \pi)\, P(U = r)$$

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Transition probabilities

Describing a model though the paradigm of the generalized DP allows to compute the so-called **transition probabilities**, i.e. **the probability of increasing one (provisional) rating point in the next step of the decision process.**

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

$$\phi_t(s) = \frac{\sum_{w_t \in d^{-1}(s)} Pr(\underline{x}(s) < X_{t+1} \leq \overline{x}(s) | W_t = w_t) Pr(W_t = w_t)}{\sum_{w_t \in d^{-1}(s)} Pr(W_t = w_t)}$$

with $t : \mathcal{D}_{W_t} \cap d^{-1}(s) \neq \emptyset$, $t < T$, where $\underline{x}(s) = \max\{d^{-1}(s)\} - w_t$ and $\overline{x}(s) = \max\{d^{-1}(s+1)\} - w_t$. In order to consider also what happens during the first step of the DP, we define $w_0 := 0$ and $\phi_0 = \phi_0(s) := Pr(\underline{x}(s) < X_1 \leq \overline{x}(s))$ with $s = d(w_0) = d(0)$.

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Describing a model though the paradigm of the generalized DP allows to compute the so-called **transition probabilities**, i.e. **the probability of increasing one (provisional) rating point in the next step of the decision process.**

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

In CUB models:

$$\phi_t(s) = 1 - \xi$$

for all *t* and *s*

Transition probabilities

Describing a model though the paradigm of the generalized DP allows to compute the so-called **transition probabilities**, i.e. **the probability of increasing one (provisional) rating point in the next step of the decision process.**

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

In NLCUB models:

$$\phi_t(s) = (1-\xi)\frac{\binom{t}{w_{g_s s}}(1-\xi)^{w_{g_s s}}\xi^{t-w_{g_s s}}}{\sum_{h=1}^{g_s}\binom{t}{w_{hs}}(1-\xi)^{w_{hs}}\xi^{t-w_{hs}}}$$

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Transition probabilities

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

$$\boxed{\phi(s) = av_t(\phi_t(s))}$$

"Perceived closeness" between rating *s* and *s* + 1

$$\delta_s = h(\phi(s))$$ for example $$\delta_s = -\log(\phi(s))$$

"Perceived distance" between rating *s* and *s* + 1

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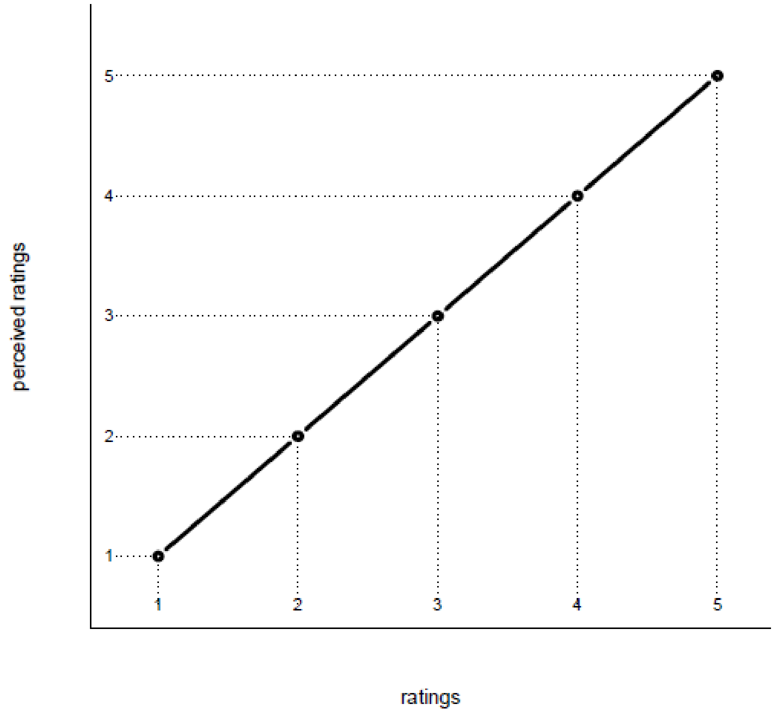
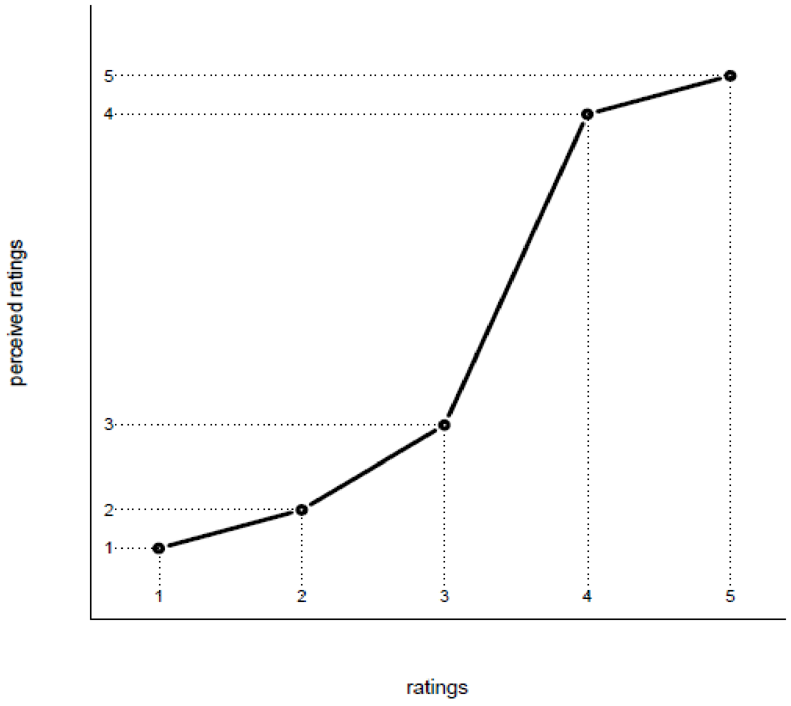
Transition plot

CUB



NLCUB (example)



Allows us to gain insight about the state of mind toward the rating scale

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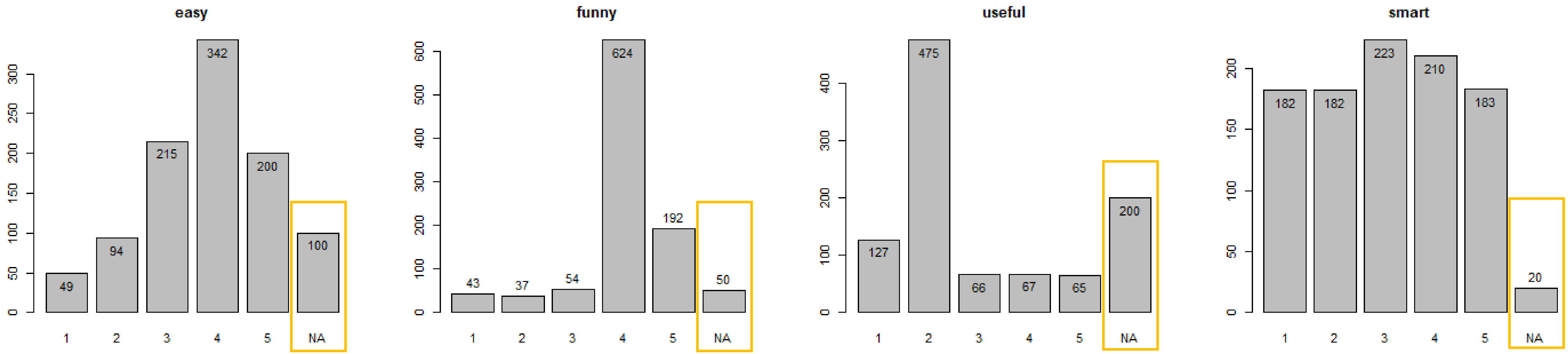
Rating data – example 1 (N=1000, 5-point Likert scale)



We show results obtained by removing NAs, but DK responses can be treated in the same way as we did with standard CUB models, by adjusting the uncertainty parameter π

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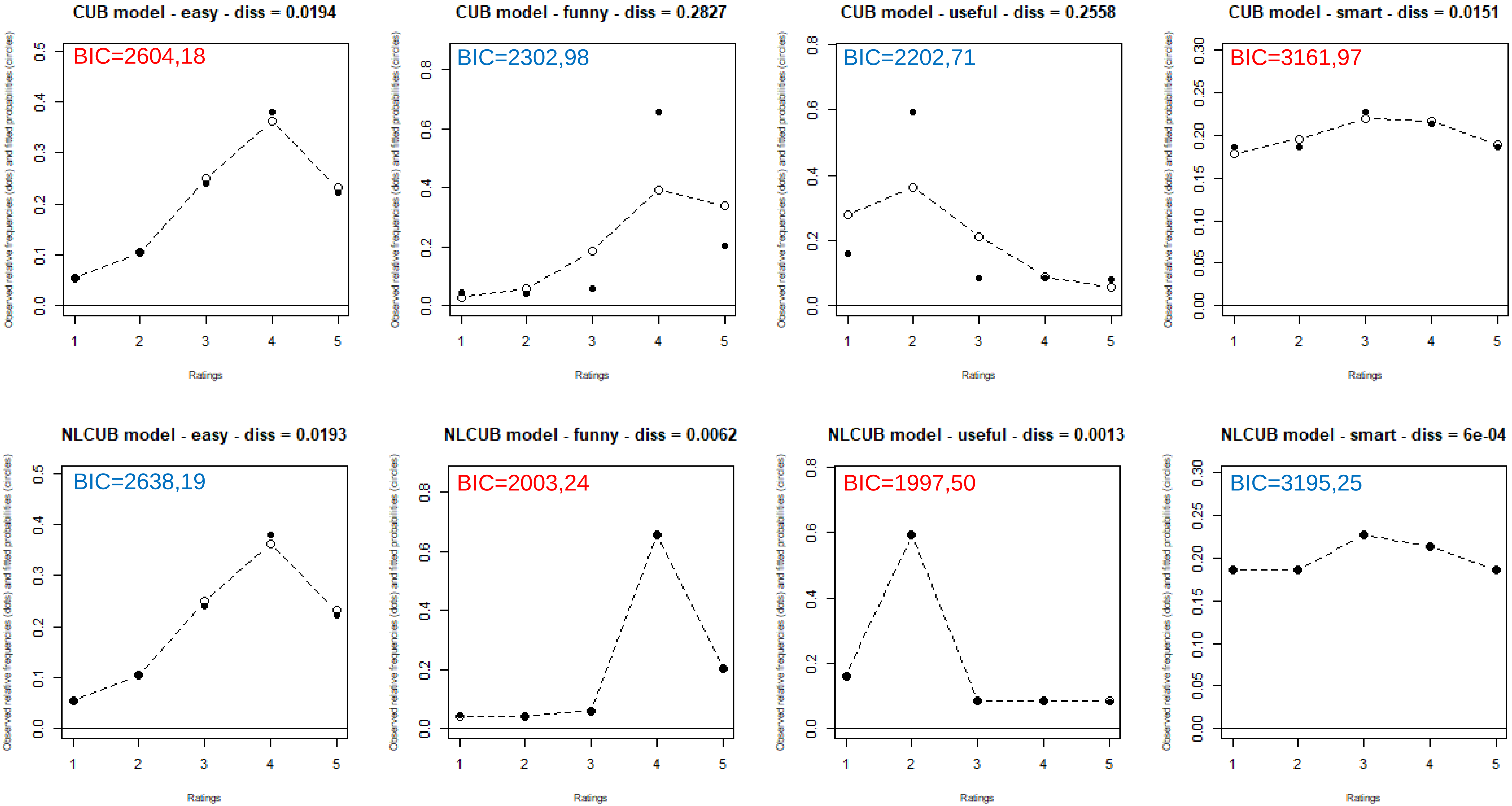
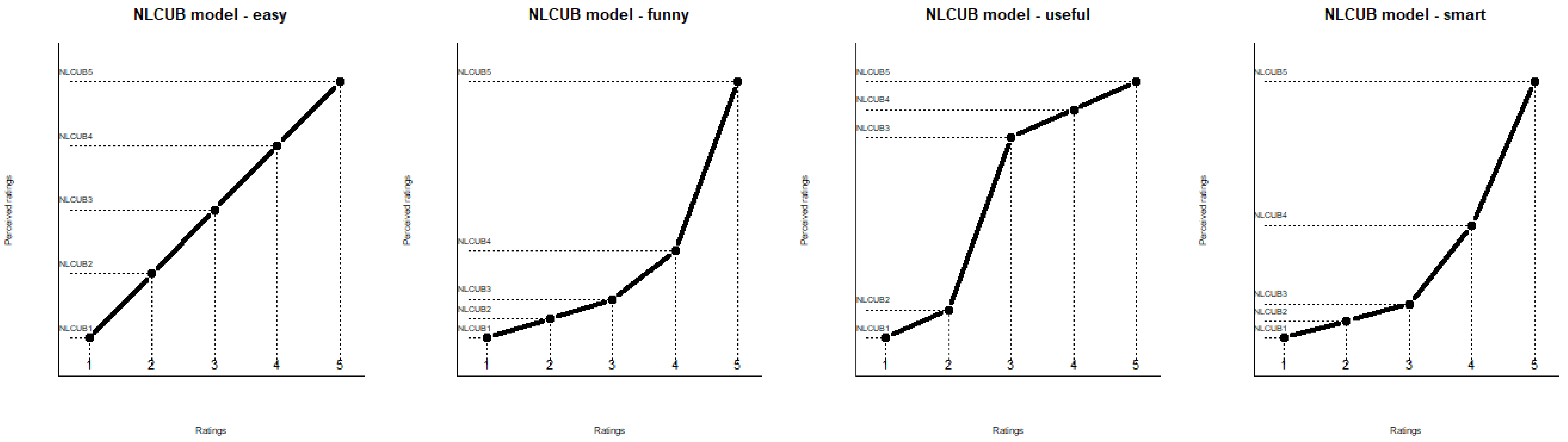

CUB model - easy - diss = 0.0194
BIC=2604,18
CUB model - funny - diss = 0.2827
BIC=2302,98
CUB model - useful - diss = 0.2558
BIC=2202,71
CUB model - smart - diss = 0.0151
BIC=3161,97
NLCUB model - easy - diss = 0.0193
BIC=2638,19
NLCUB model - funny - diss = 0.0062
BIC=2003,24
NLCUB model - useful - diss = 0.0013
BIC=1997,50
NLCUB model - smart - diss = 6e-04
BIC=3195,25
Observed relative frequencies (dots) and fitted probabilities (circles)
Ratings

Rating data – example 1 (N=1000, 5-point Likert scale)



Note that NLCUB includes standard CUB as a special case

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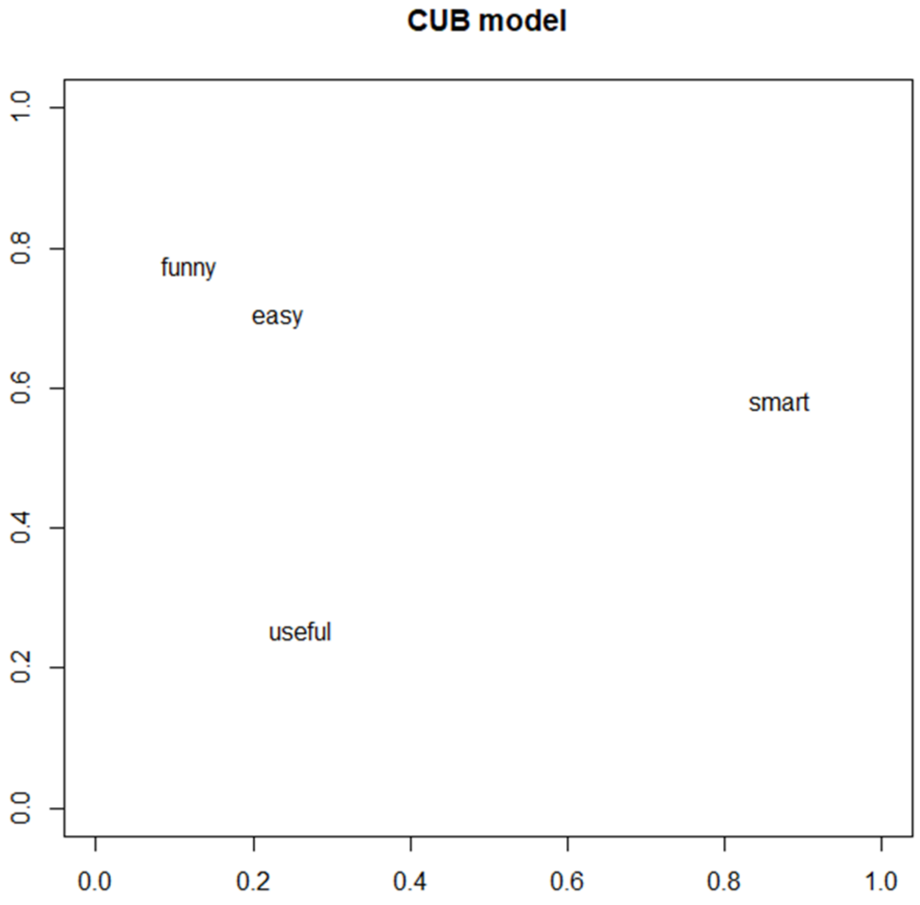
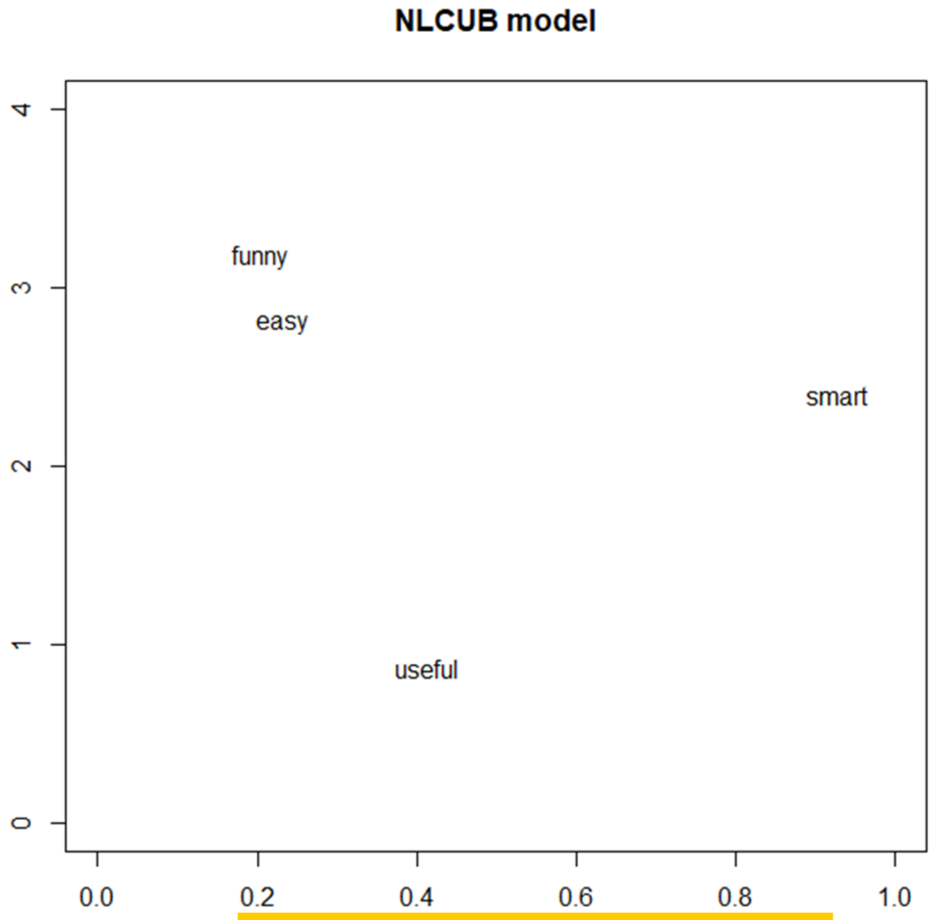


Feeling parameter: $1 - \xi$

Uncertainty parameter: $1 - \pi$



Feeling parameter: μ

Uncertainty parameter: $1 - \pi$

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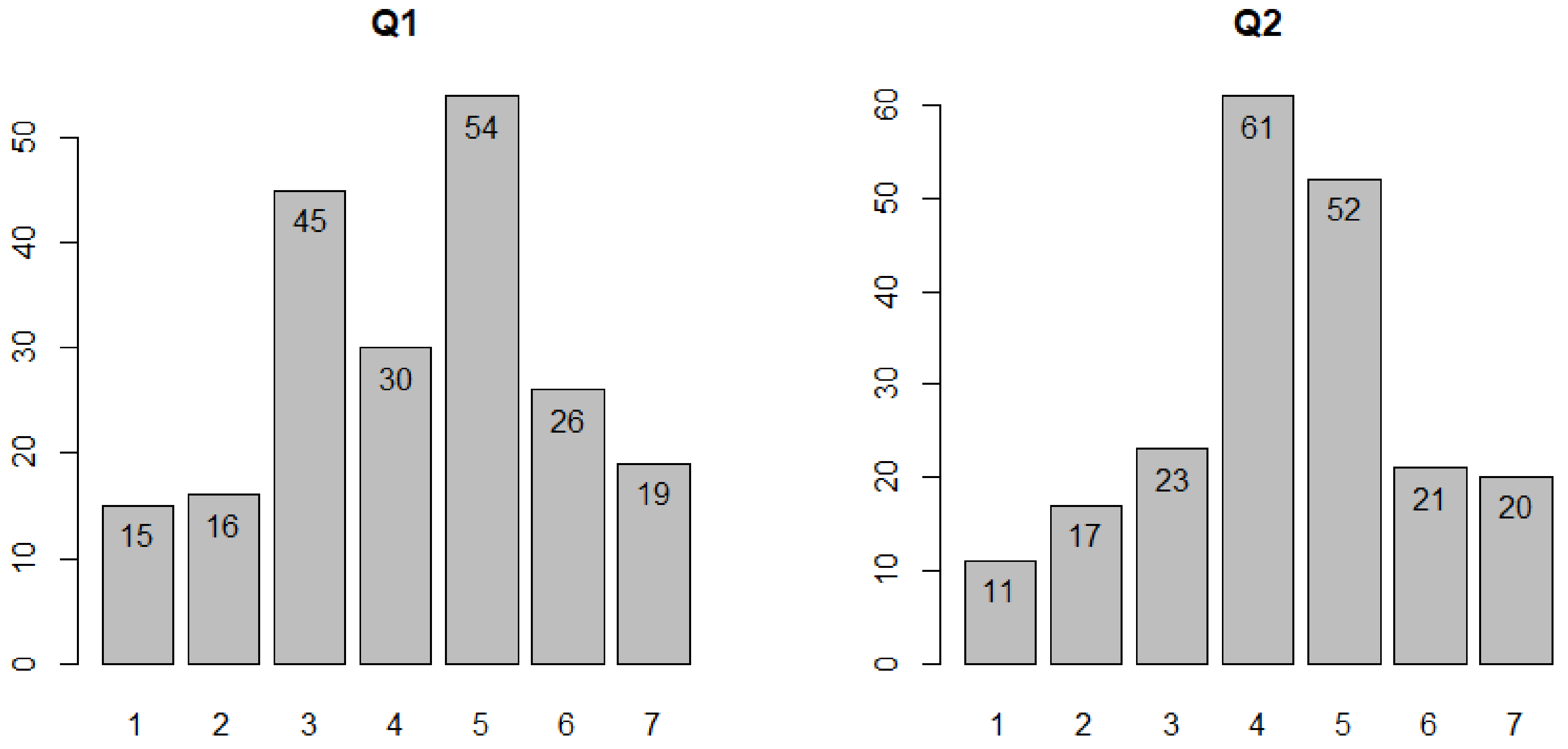
Rating data – example 2 (N=205, 7-point semantic differential scale)

Please rate your opinion about the following questions between the two given extreme options

- Q1: Do you think that your learning was improved or worsened by distance teaching? (1=Much worsened, 4=Neither worsened nor improved, 7=Much improved)
- Q2: Do you think that the solutions arranged for distance teaching should be maintained or abandoned after the COVID-19 pandemic? (1=We should go completely back to the past, 4=We should select what to maintain and what to abandon, 7=We should maintain all the novelties introduced during the pandemic)

Rating data – example 2 (N=205, 7-point semantic differential scale)

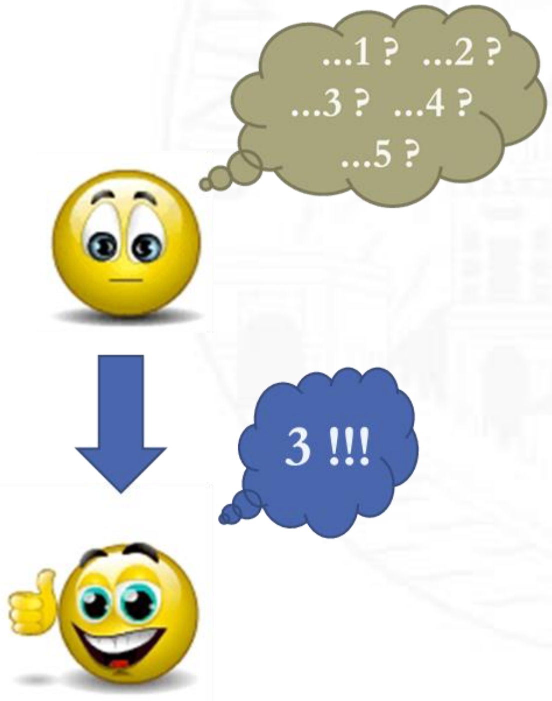


The unconscious Decision Process (DP)

CUM (Combination of Uniform and Multinomial) model

- <u>the **distribution of the basic judgments**</u>: k (where $m = 2k + 1$) **Multinoulli** random variables where **three basic sensations are possible**: a **positive** one (with probability ξ_U, which will drive an upward movement along the rating scale), a **negative** one (with probability ξ_D, which will drive a downward movement along the rating scale) and a **neutral** one (with probability $1 - \xi_U - \xi_D$, which will involve staying still in the rating scale).

Please rate your opinion about the following questions between the two given extreme options

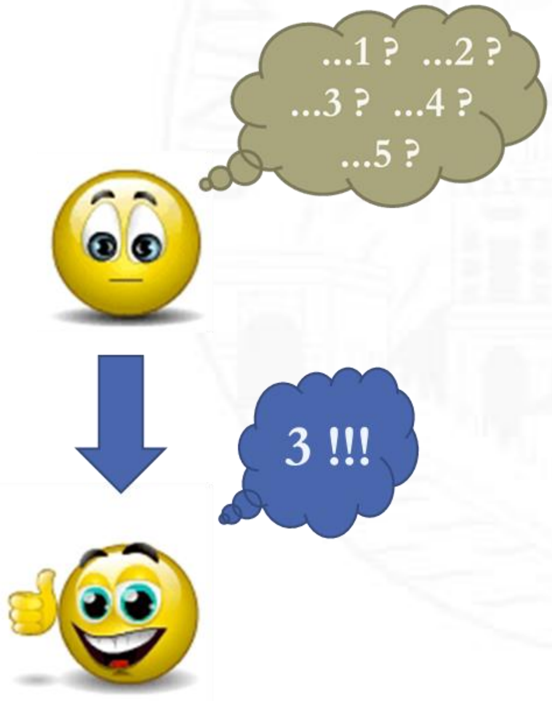


The unconscious Decision Process (DP)

CUM (Combination of Uniform and Multinomial) model

- <u>the **accumulation function**</u>: **sum** (which generates a **Multinomial** random variable). At each step the number of positive, negative and neutral basic judgments is considered.

Please rate your opinion about the following questions between the two given extreme options

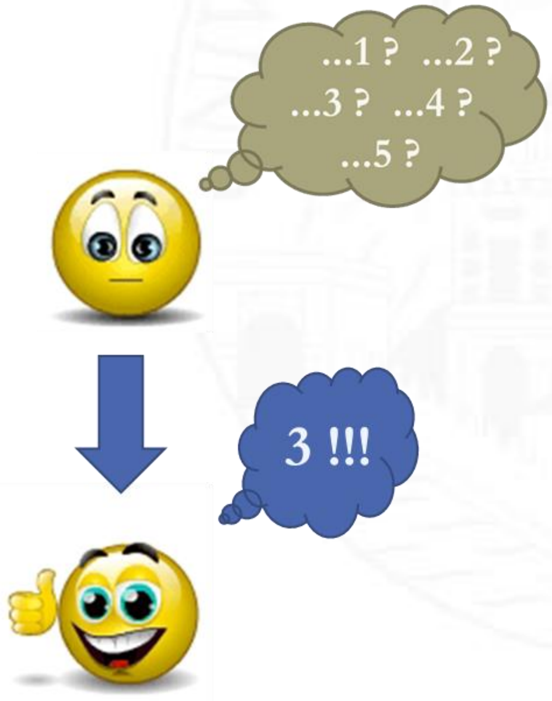


The unconscious Decision Process (DP)

CUM (Combination of Uniform and Multinomial) model

- **the transformation function**: a linear transformation of the Multinomial random variable onto the scale 1, 2, …, *m*. The transformation is done by impressing an upward (downward) movement in the rating scale for each positive (negative) basic judgment.

Please rate your opinion about the following questions between the two given extreme options



The unconscious Decision Process (DP)

Do you agree with ABC?

Express a rating from 1 (=totally disagree) to 5 (=totally agree)

Two feeling parameters

Feeling approach CUM: linearly transformed Multinomial random variable (W)

$$P(W = r) = W(r|\xi_D, \xi_U)$$

Uncertainty approach CUM: Uniform random variable (U)

$$P(U = r) = 1/m$$

Expressed rating NLCUB: mixture of A and U (R)

$$P(R = r|\theta) = \pi P(W = r) + (1 - \pi) P(U = r)$$

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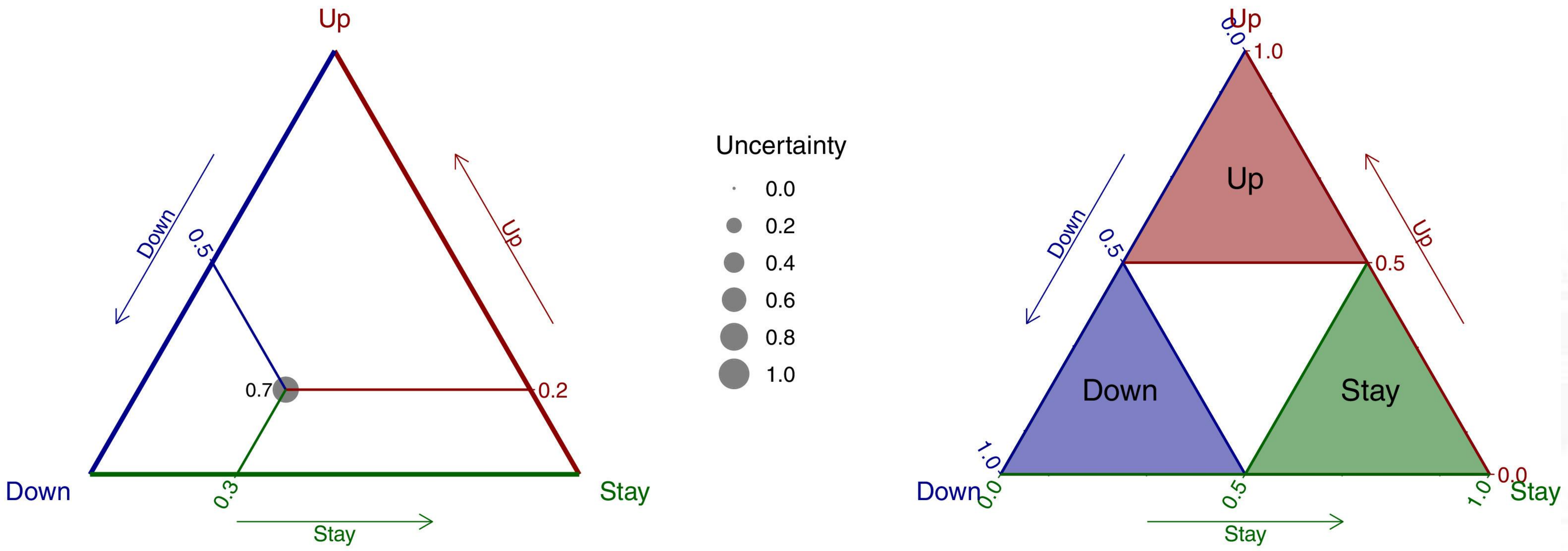
Graphical representation of the parameter space



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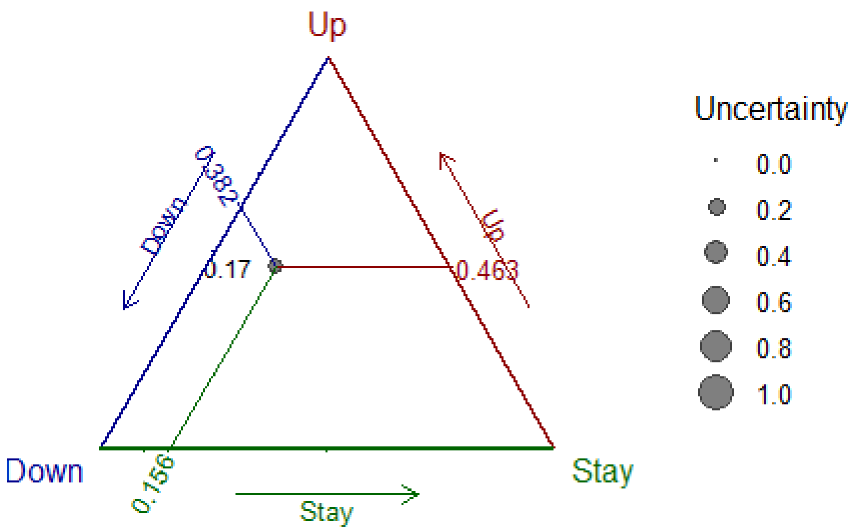
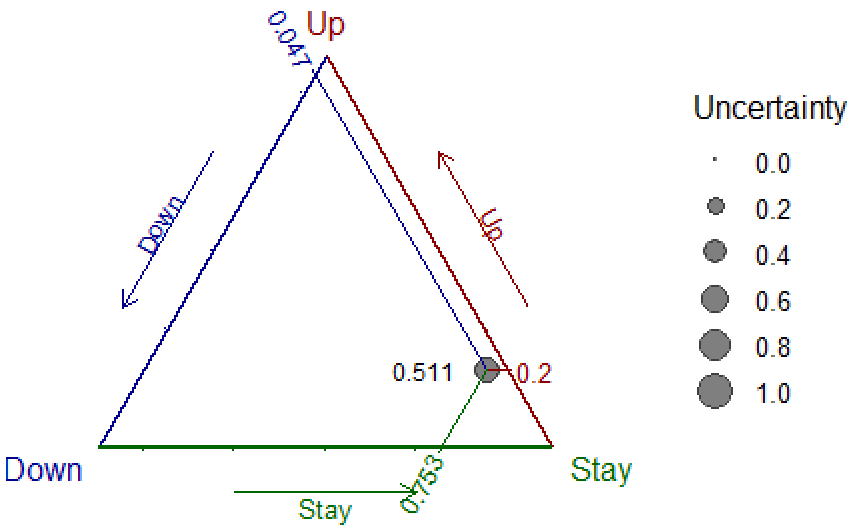
Rating data – example 2 (N=205, 7-point semantic differential scale)





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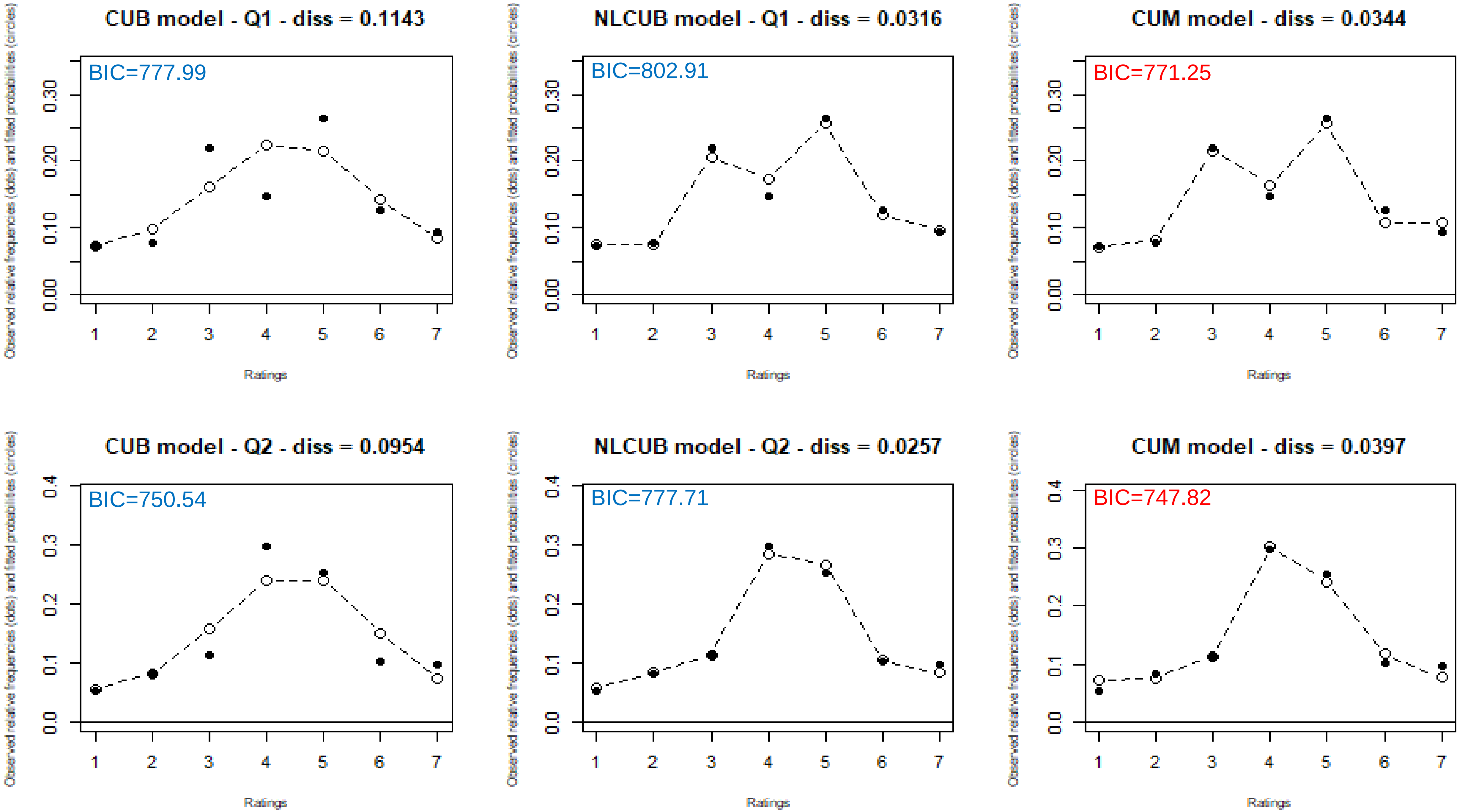
CUB model - Q1 - diss = 0.1143
BIC=777.99
NLCUB model - Q1 - diss = 0.0316
BIC=802.91
CUM model - diss = 0.0344
BIC=771.25
CUB model - Q2 - diss = 0.0954
BIC=750.54
NLCUB model - Q2 - diss = 0.0257
BIC=777.71
CUM model - diss = 0.0397
BIC=747.82
Ratings
Observed relative frequencies (dots) and fitted probabilities (circles)

R functions and codes

All the examples shown in these slides can be reproduced by downloading data, scripts and functions at the page https://bodai.unibs.it/cub/

Main references (others can be found at https://bodai.unibs.it/cub/)

D'Elia A., Piccolo D. (2005), A mixture model for preferences data analysis, Computational Statistics & Data Analysis, **49**(3), 917-934.

Manisera M., Zuccolotto P. (2014), Modeling rating data with Nonlinear CUB models, *Computational Statistics & Data Analysis*, **78**, 100-118.

Manisera M., Zuccolotto P. (2014), Modeling "don't know" responses in rating scales, *Pattern Recognition Letters*, **45**, 226-234.

Manisera M., Zuccolotto P. (2022), A mixture model for ordinal variables measured on semantic differential scales, *Econometrics and Statistics*, **22**, 98-123.

Piccolo, D., Simone, R. (2019). The class of cub models: statistical foundations, inferential issues and empirical evidence. *Statistical Methods & Applications*, **28**, 389-435.