Nonlinear CUB models

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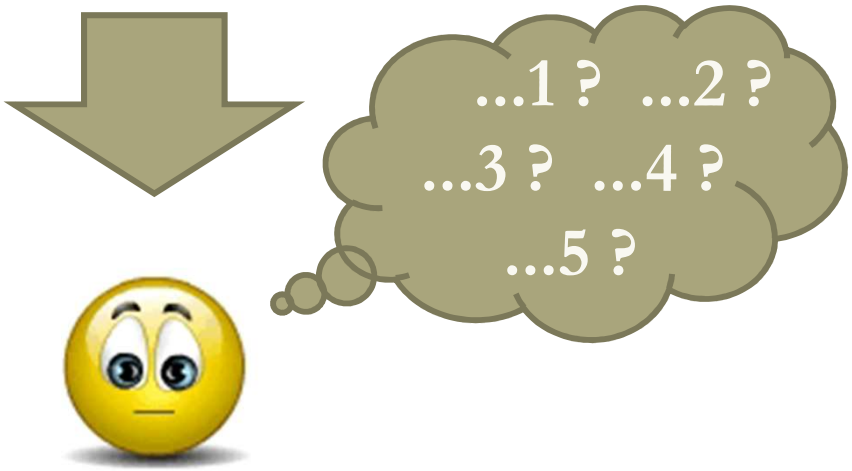
Agenda

- The unconscious Decision Process (DP) driving individuals' responses on a rating scale
- Focus: the Feeling phase of the DP in CUB and NLCUB models
- Transition Probabilities and Transition Plot
- Linear and Nonlinear DPs
- NLCUB
- Case study: Standard Eurobarometer 78

(SYRTO: "SYstemic Risk TOmography: Signals, Measurements, Transmission Channels, and Policy Interventions", grant from the European Union Seventh Framework Programme - Project ID: 320270)

Are you satisfied with XYZ?

Express a rating from 1 (=completely unsatisfied) to 5 (=completely satisfied)



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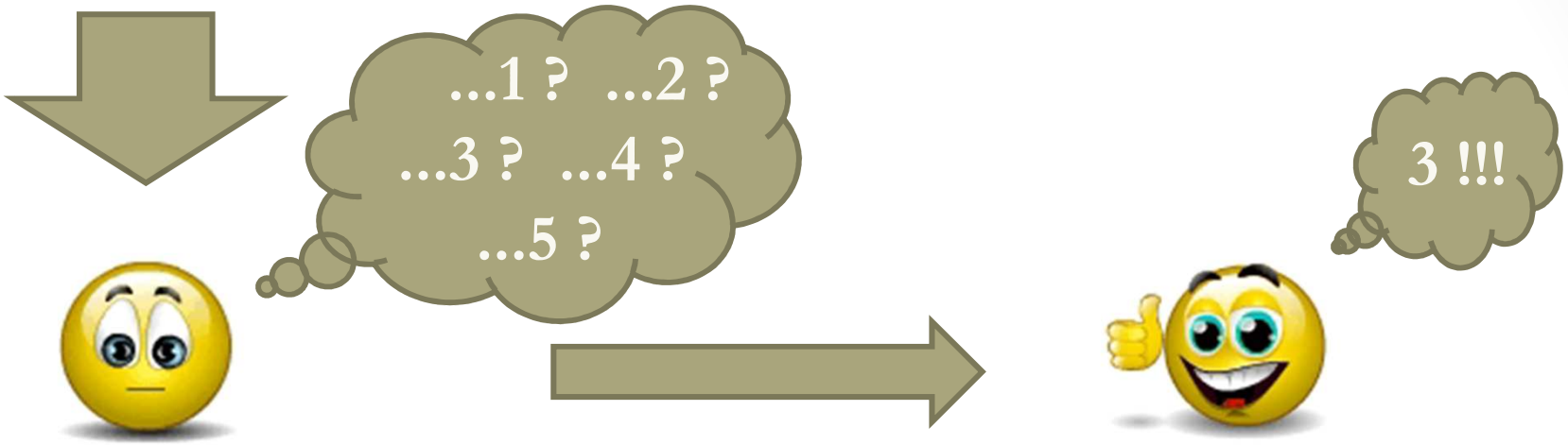

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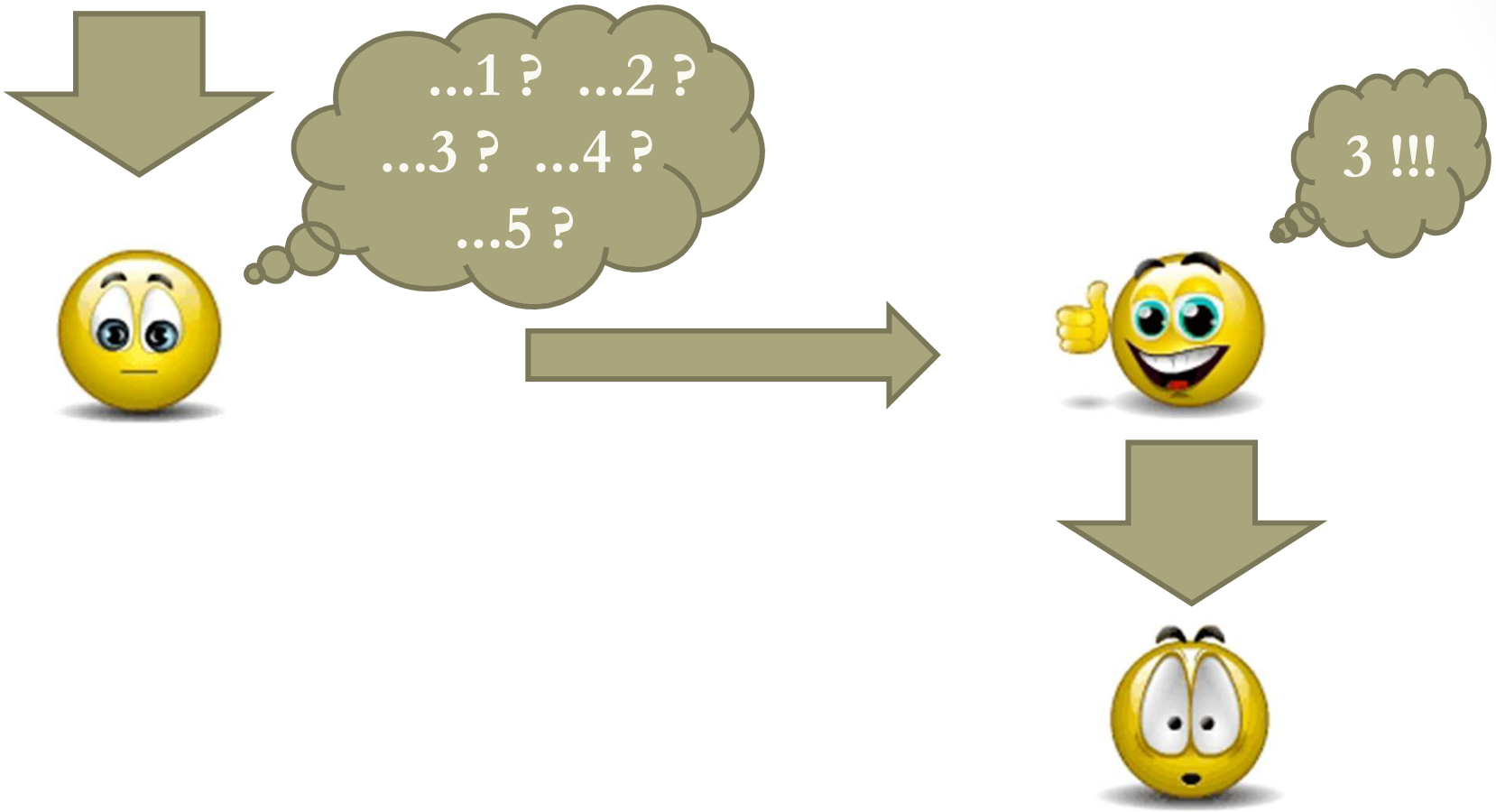

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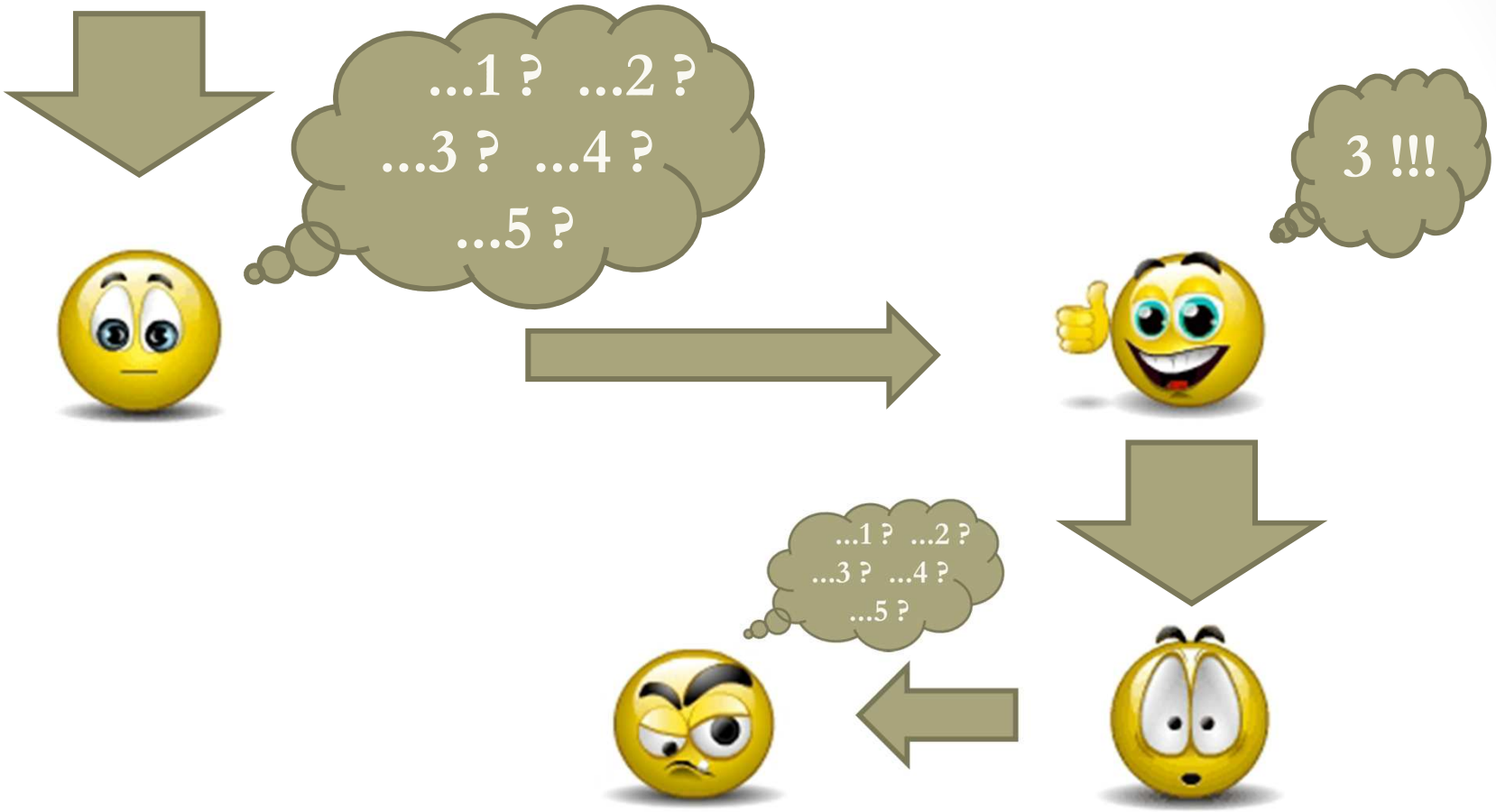

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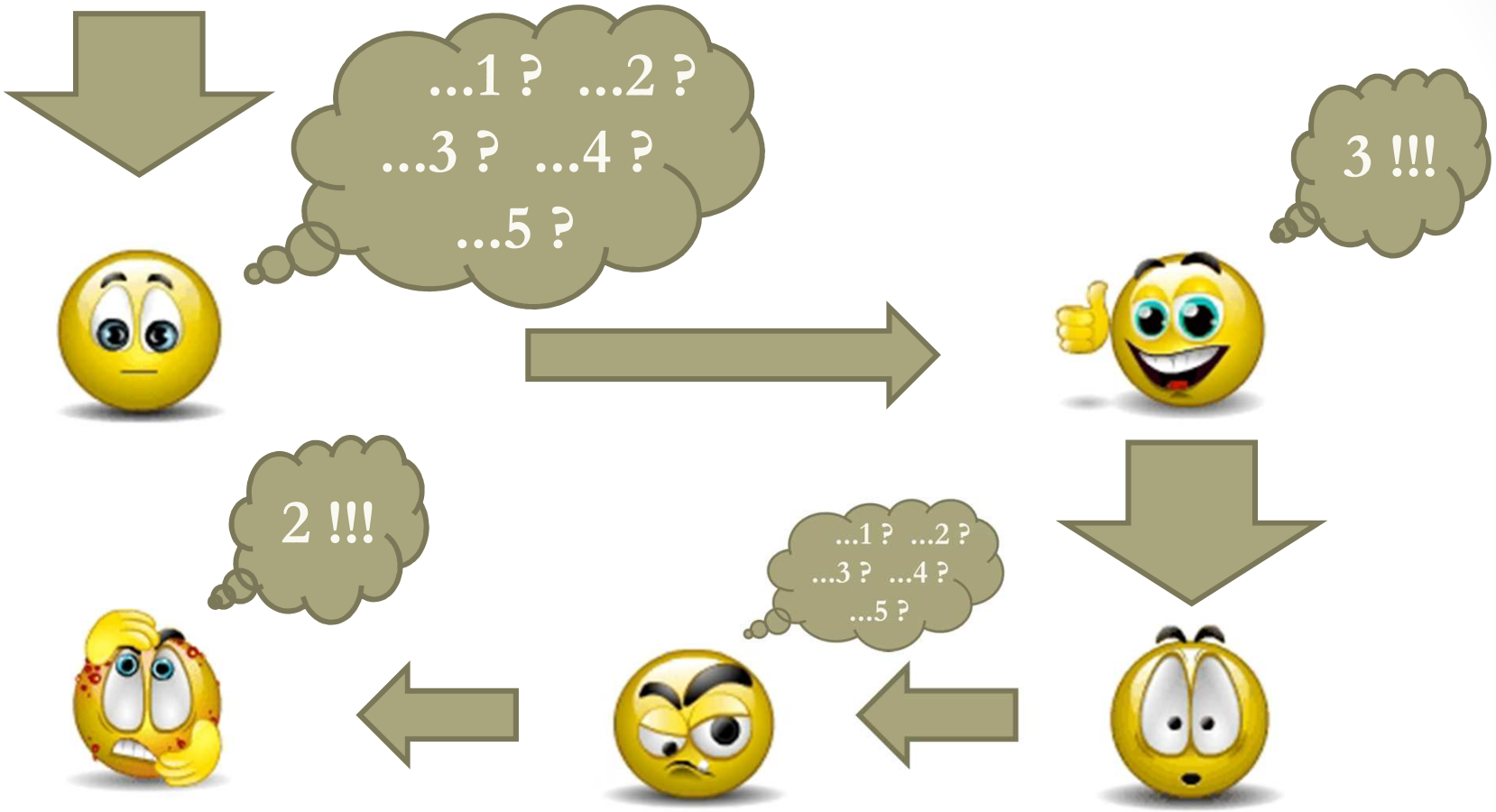

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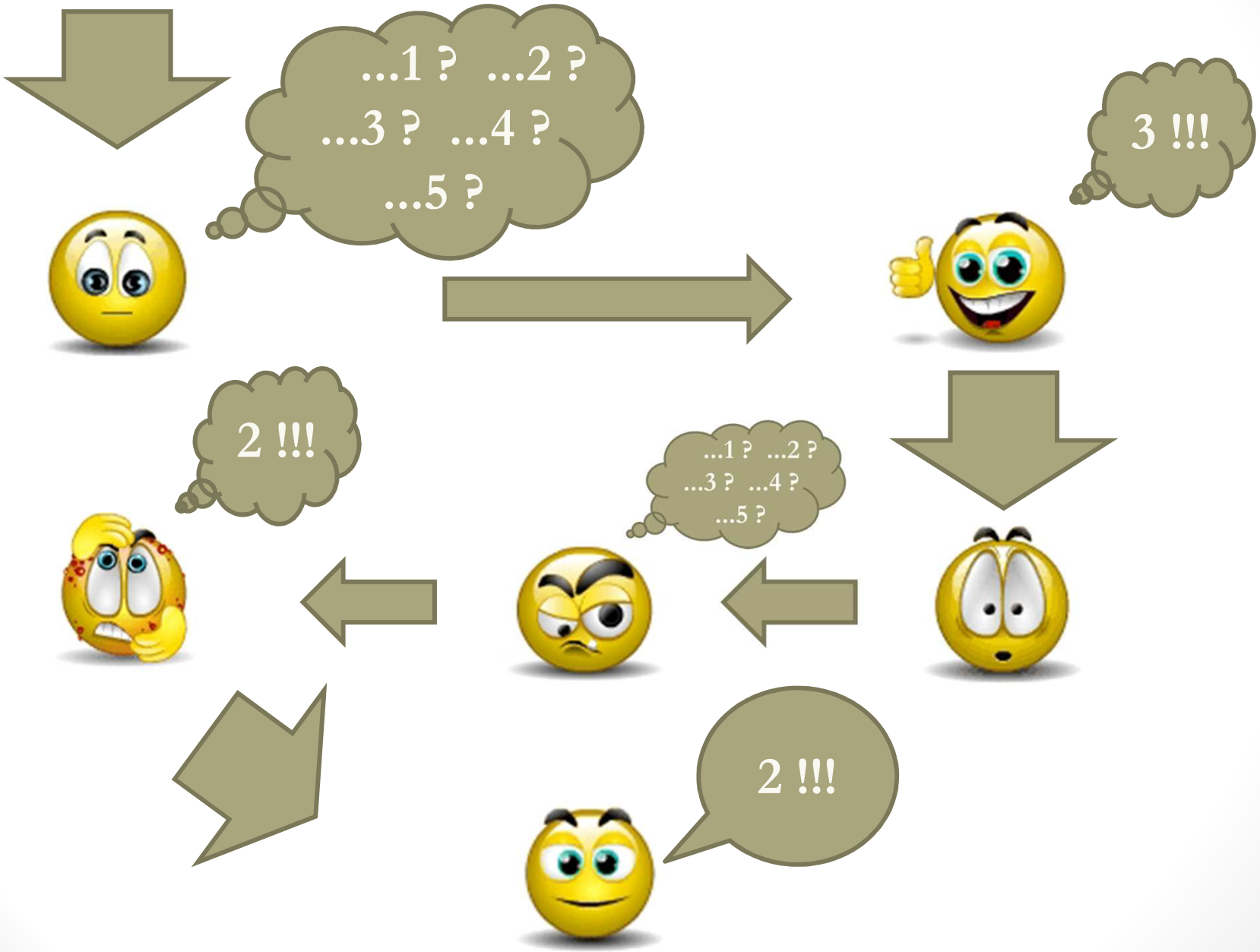

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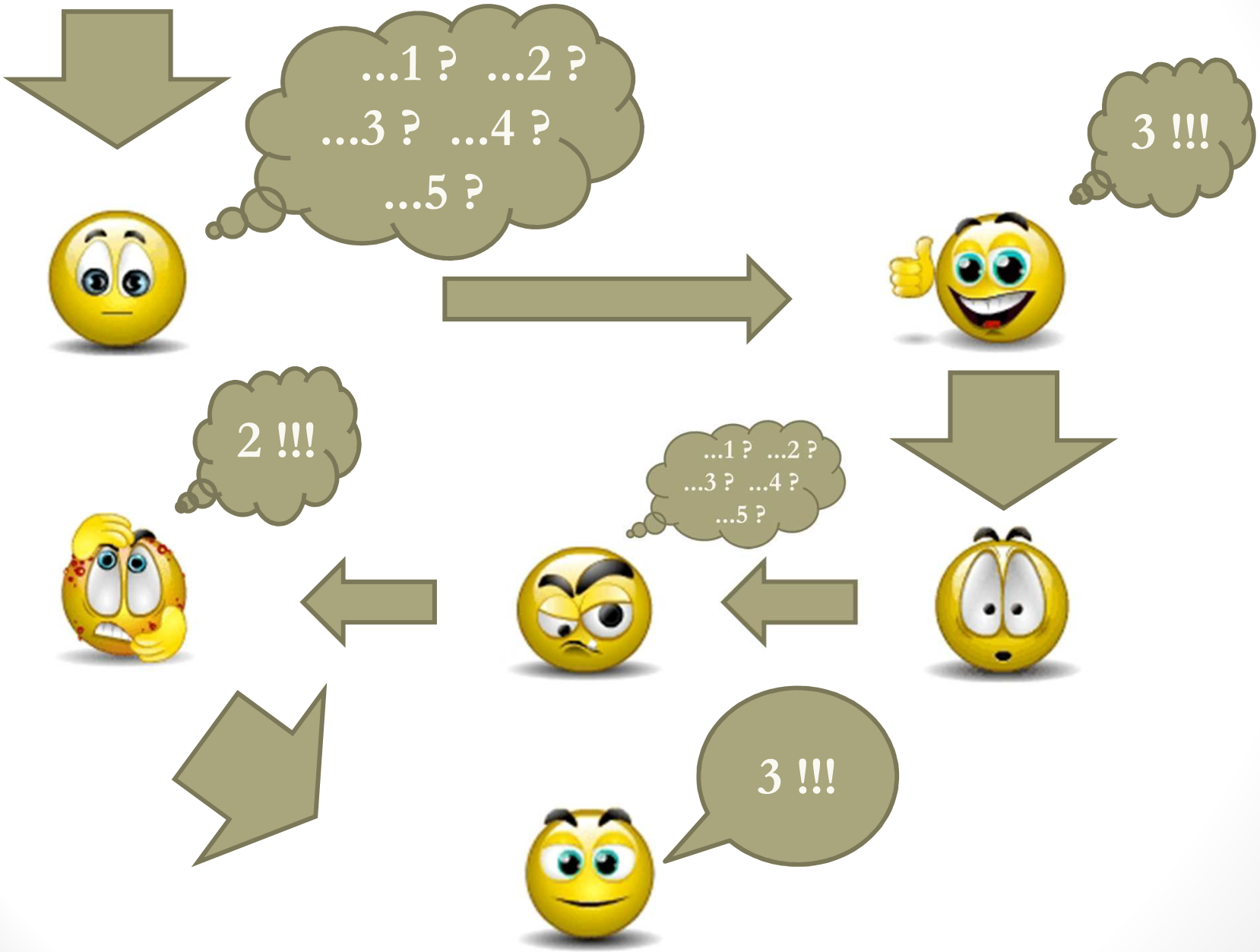

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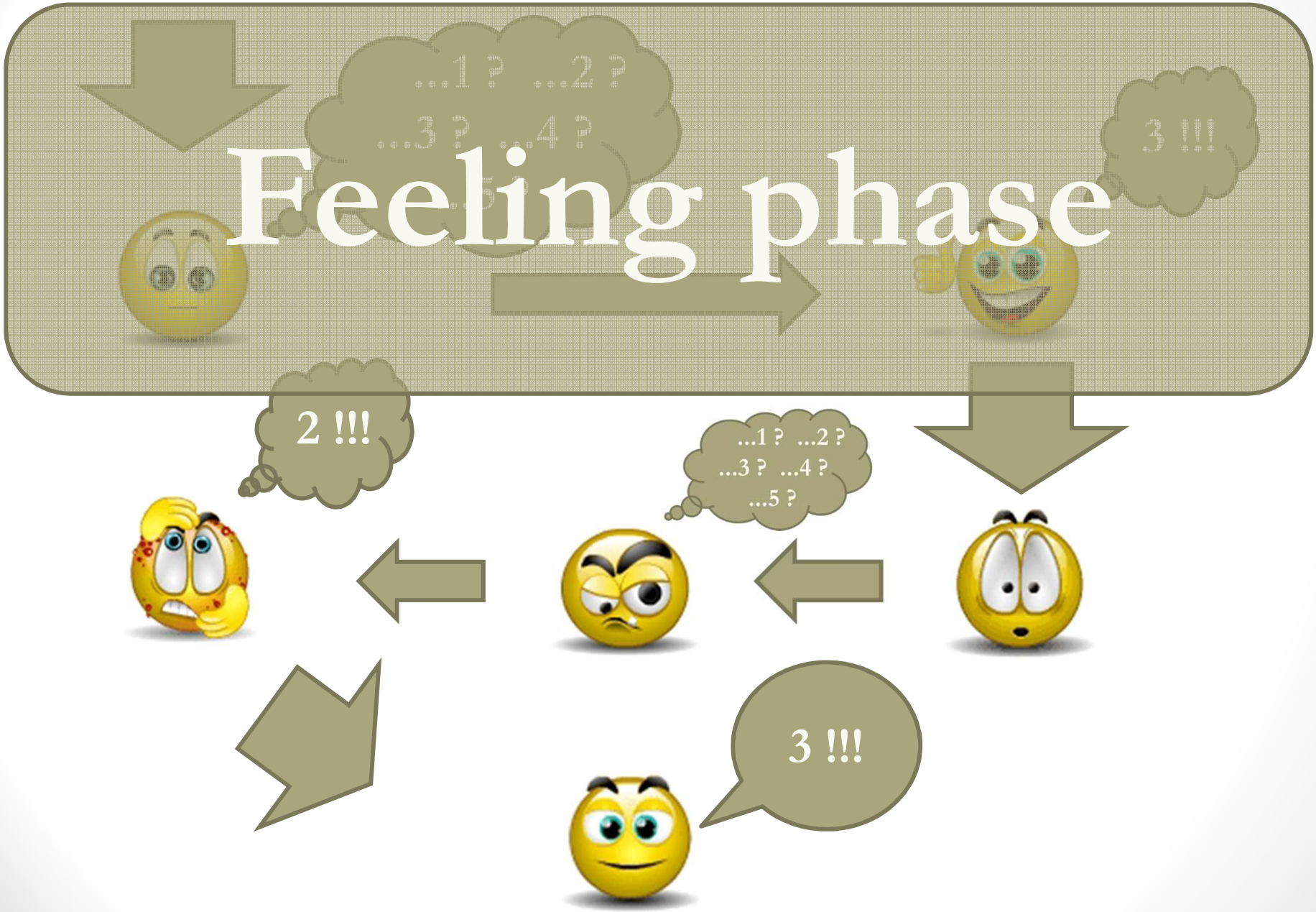

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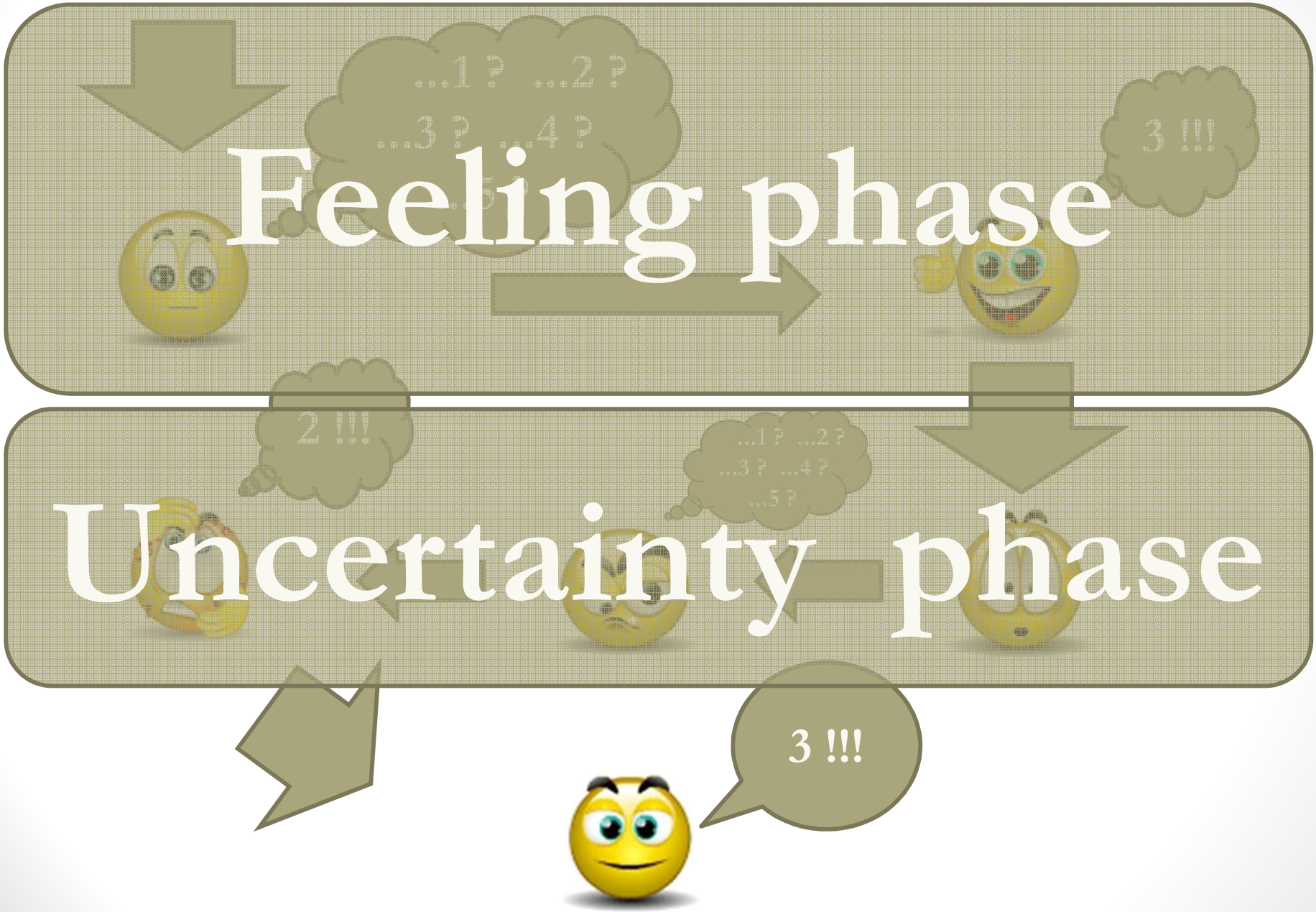

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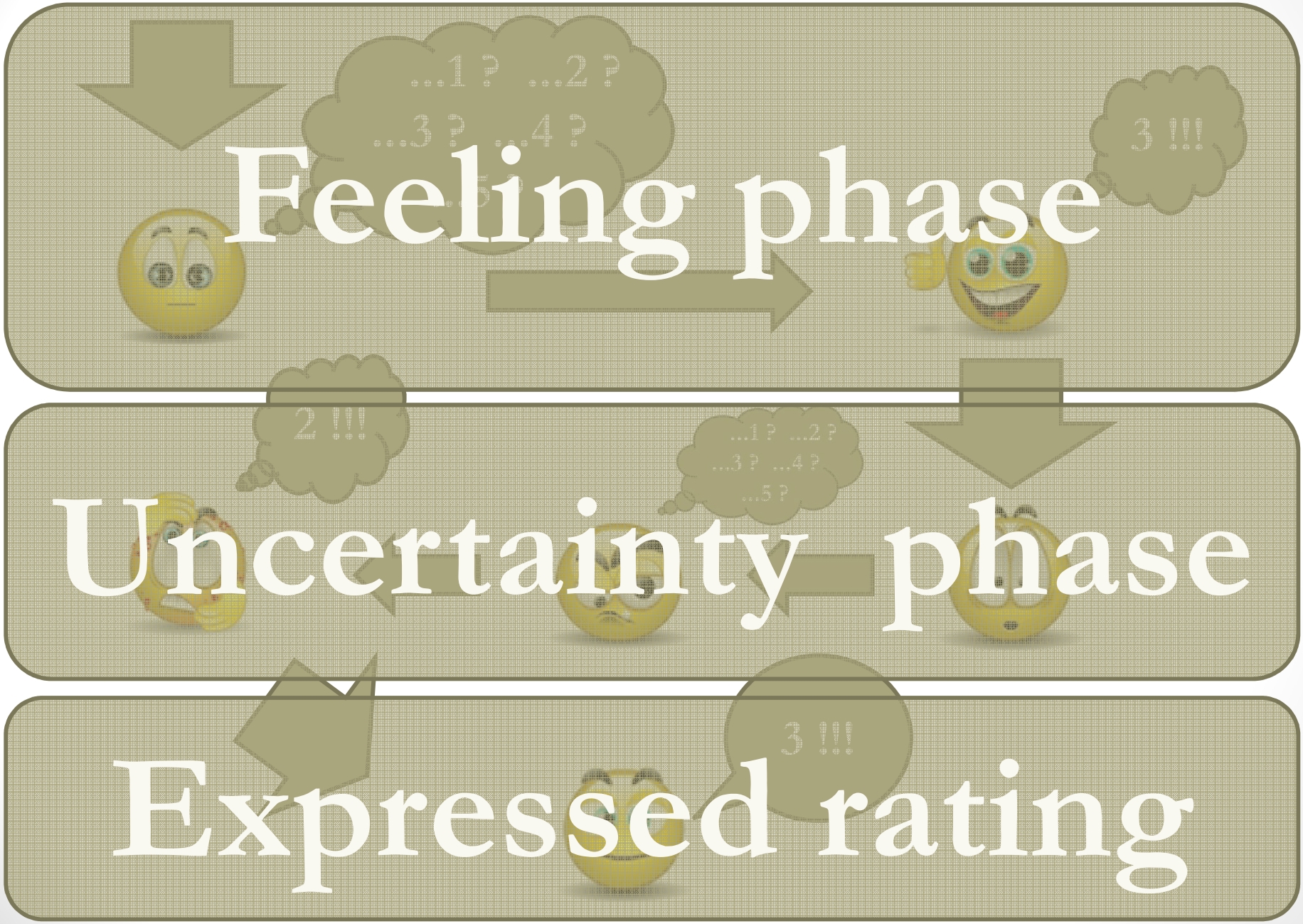

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Feeling phase:

reasoned and logical thinking, the set of emotions, perceptions, subjective evaluations that individuals have with regard to the latent trait being evaluated

Uncertainty phase

Expressed rating

Are you satisfied with XYZ?

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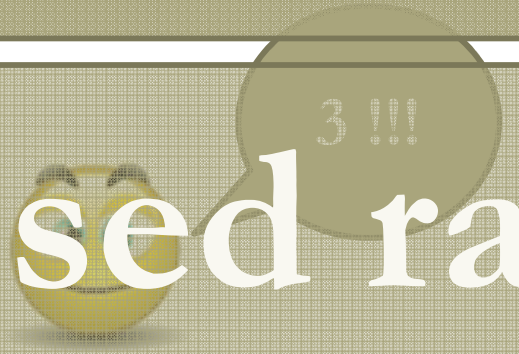

Expressed rating

The unconscious DP

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Feeling phase:
reasoned and logical thinking, the set of emotions, perceptions, subjective evaluations that individuals have with regard to the latent trait being evaluated

Uncertainty phase: ***indecision inherently present in any human choice, not depending on the individuals' position on the latent variable***

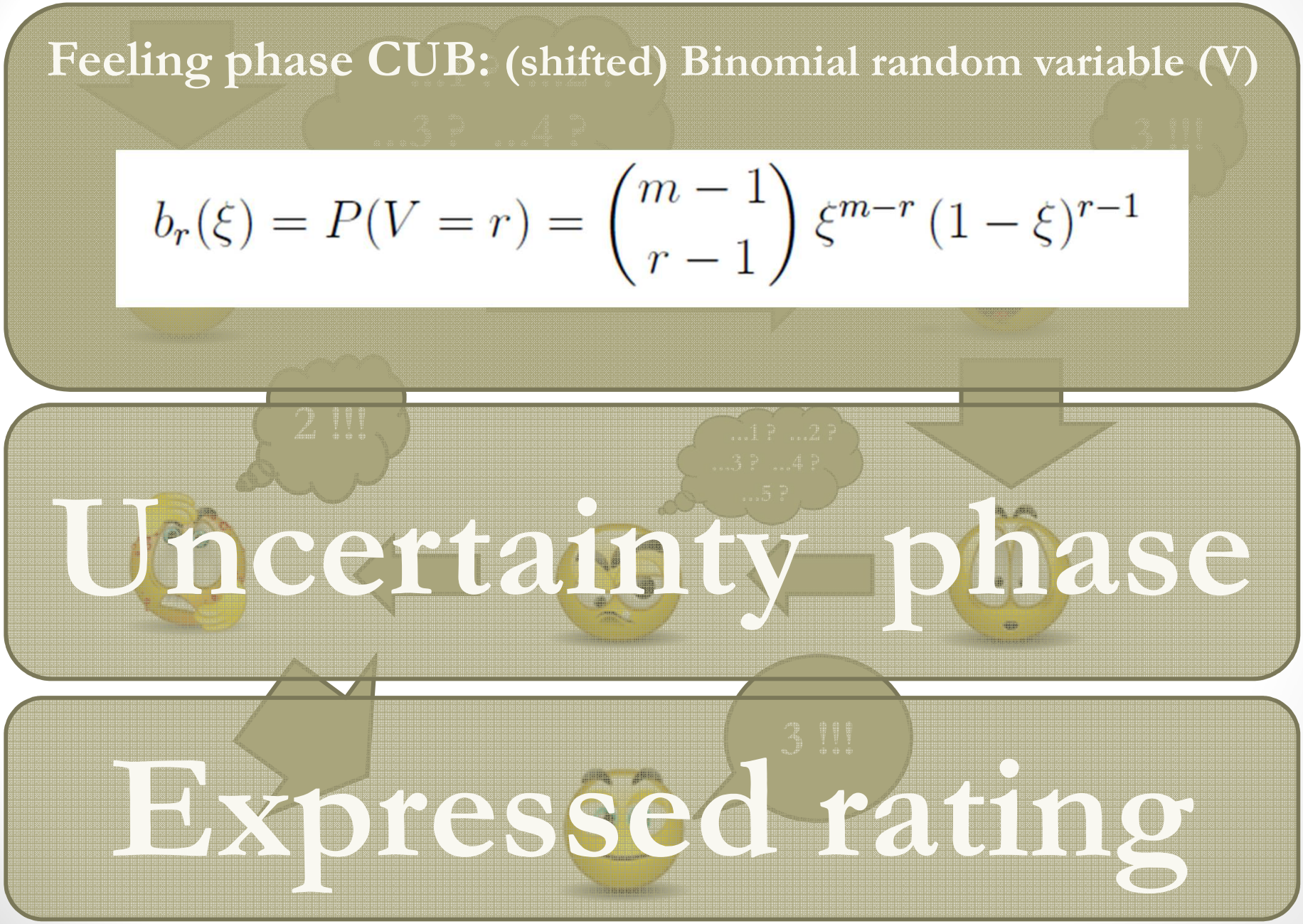
Expressed rating: ***a combination of the first two phases***

Are you satisfied with XYZ?

Express a rating from 1 (=completely unsatisfied) to 5 (=completely satisfied)

Feeling phase CUB: (shifted) Binomial random variable (V)

$$b_r(\xi) = P(V = r) = \binom{m-1}{r-1} \xi^{m-r} (1-\xi)^{r-1}$$



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Uncertainty phase CUB: Uniform random variable (U)

$$P(U = r) = 1/m$$

Expressed rating

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Uncertainty phase CUB: Uniform random variable (U)

$$P(U = r) = 1/m$$

Expressed rating CUB: mixture of V and U (R)

$$P(R = r \mid \boldsymbol{\theta}) = \pi\, b_r(\xi) \;+\; (1-\pi)\, P(U = r)$$

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Express a rating from 1 (=completely unsatisfied) to 5 (=completely satisfied)

Feeling phase:

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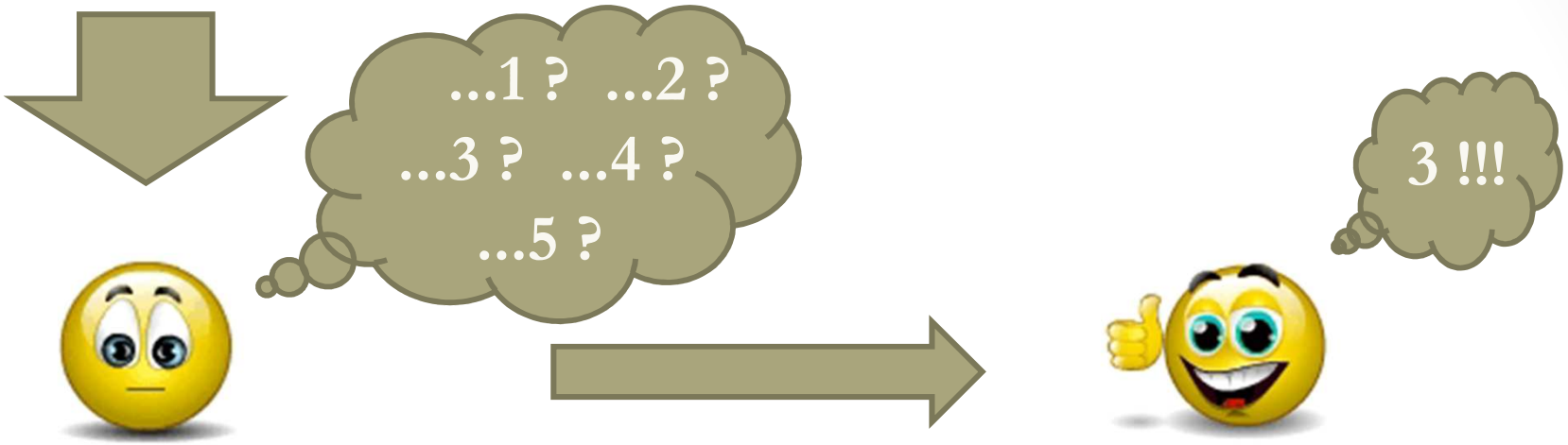

- We assume that the Feeling phase proceeds through T consecutive steps.
- At each step a basic judgement is formulated.
- Step-by-step, the basic judgements are accumulated and transformed into provisional ratings.
- The last provisional rating corresponds to the rating at the end of the Feeling phase.

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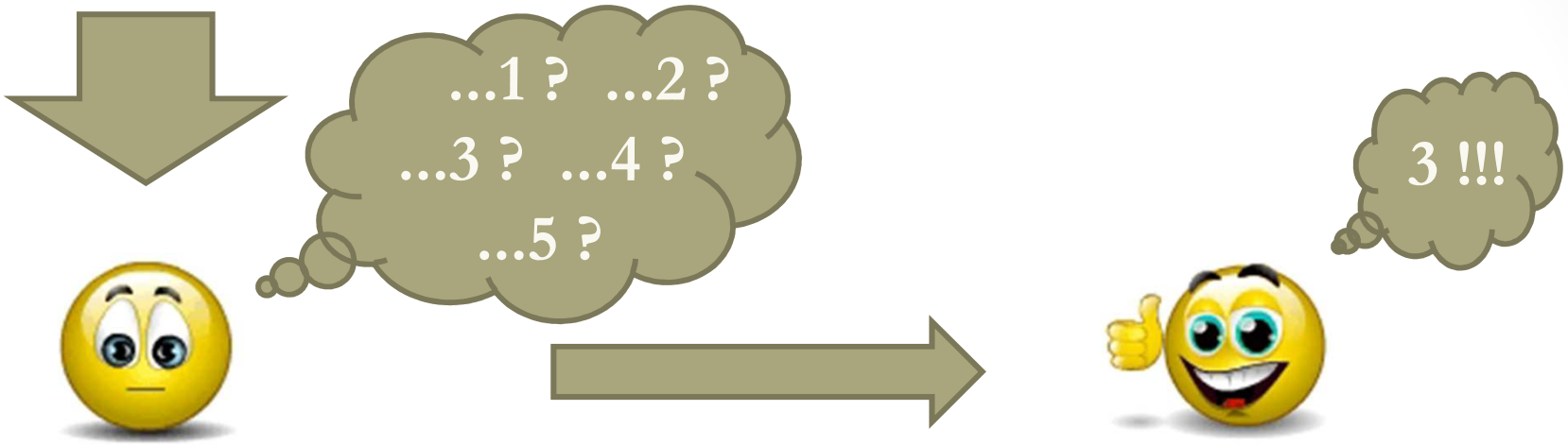

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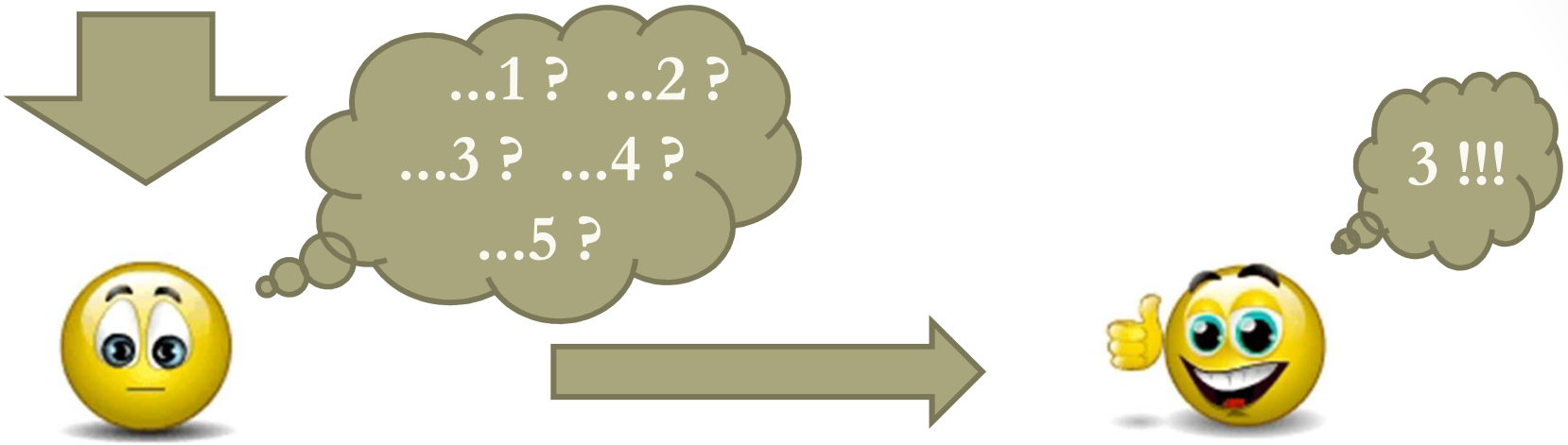

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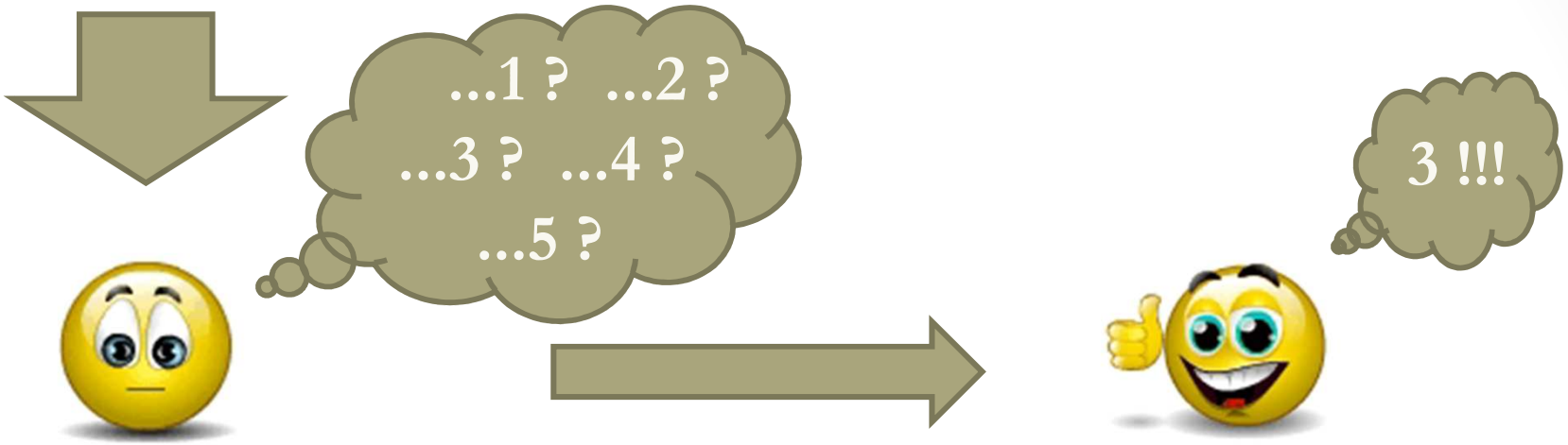

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What happens in a CUB model?

$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

Step	Basic judgement
1	
2	
3	
4	

What happens in a CUB model?

$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

Step	Basic judgement
1	
2	
3	
4	

What happens in a CUB model?

$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

Step	Basic judgement	Accumulated judgement (# of Yes)
1		0
2		1
3		1
4		2

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$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

Step	Basic judgement	Accumulated judgement (# of Yes)
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2		1
3		1
4		2

rule

Provisional rating
1
2
2
3

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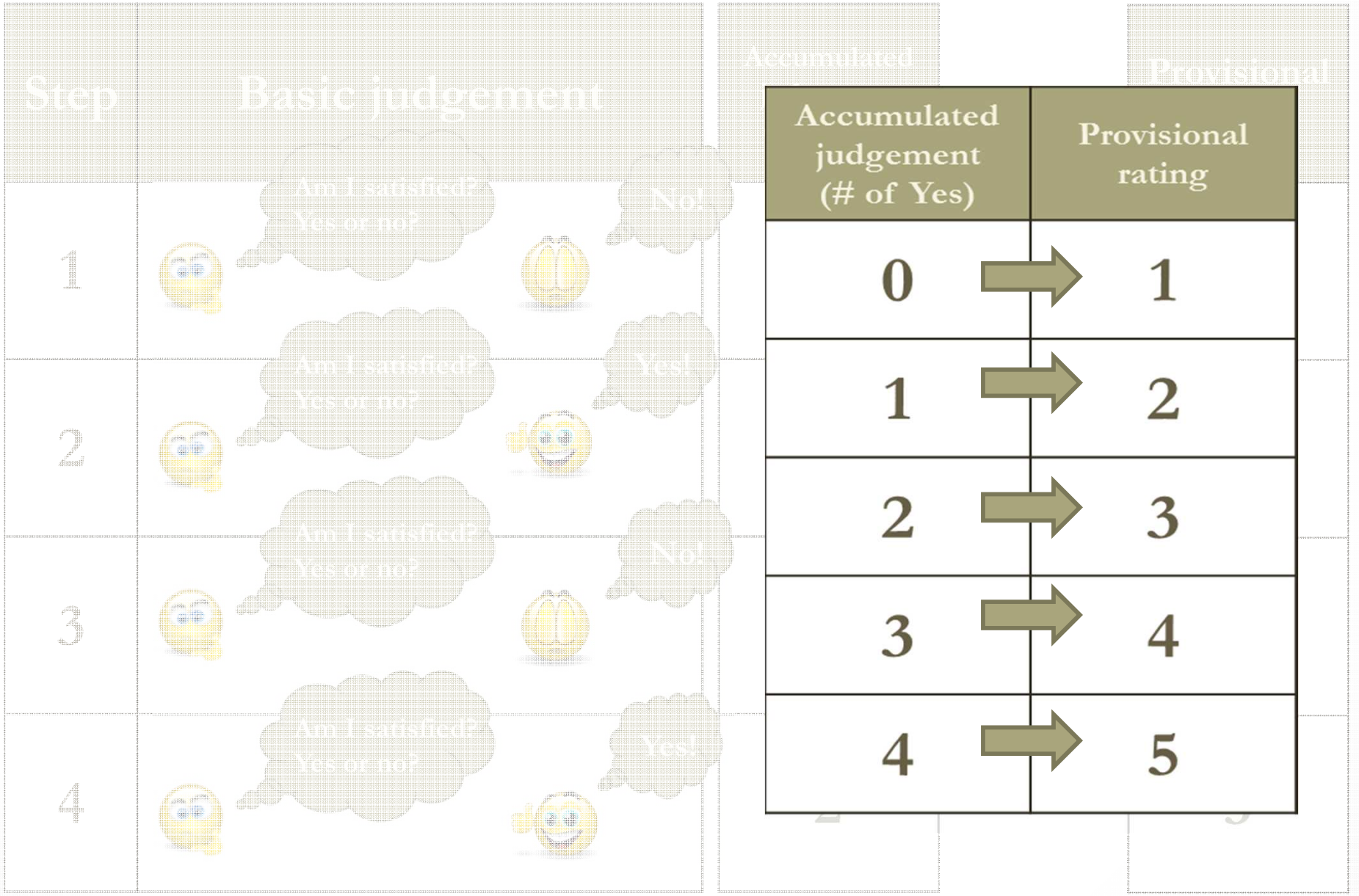


Accumulated judgement (# of Yes)	Provisional rating
0	1
1	2
2	3
3	4
4	5

What happens in a CUB model?

$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

Step	Basic judgement	Accumulated judgement (# of Yes)
1	Am I satisfied? Yes or no? / No!	0
2	Am I satisfied? Yes or no? / Yes!	1
3	Am I satisfied? Yes or no? / No!	1
4	Am I satisfied? Yes or no? / Yes!	2

rule

Provisional rating
1
2
3

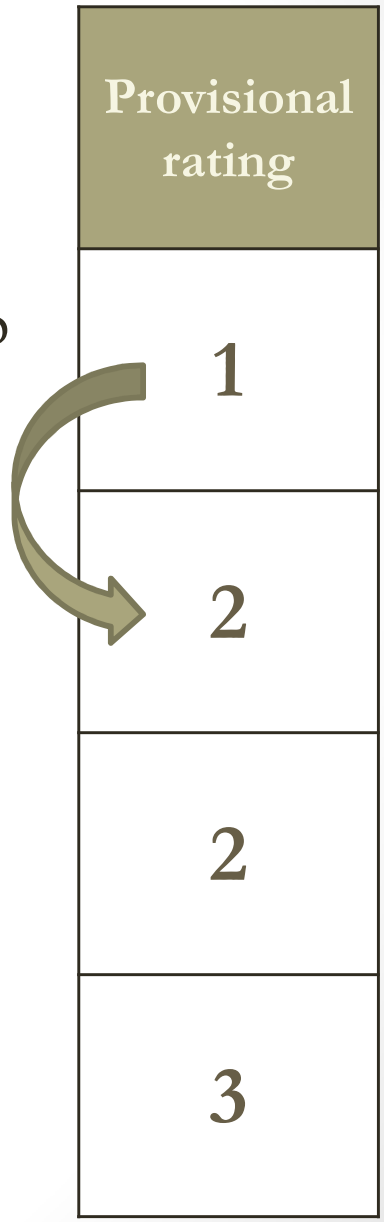
3 !!!

What happens in a CUB model?

$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

TRANSITION PROBABILITIES

Probability to move to provisional rating **2** in the next step given that the provisional rating in the present step is **1**



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TRANSITION PROBABILITIES

Probability to move to provisional rating **2** in the next step given that the provisional rating in the present step is **1**

Probability to move to provisional rating **3** in the next step given that the provisional rating in the present step is **2**

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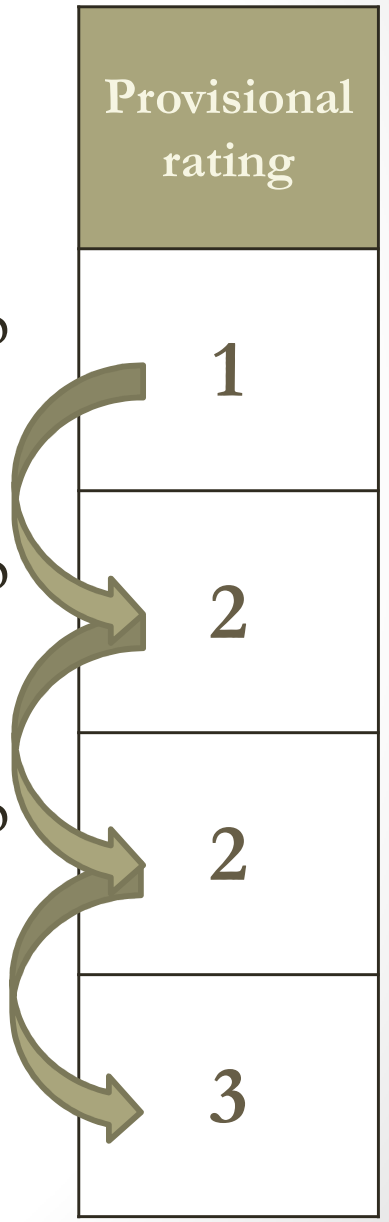


What happens in a CUB model?

$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

TRANSITION PROBABILITIES

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

R_t
1
2
2
3

What happens in a CUB model?

$T = m - 1$ steps in the Feeling phase (in this example $T = 4$)

TRANSITION PROBABILITIES

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

In CUB models:

$$\phi_t(s) = 1 - \xi$$

for all t and s

R_t
1
2
2
3

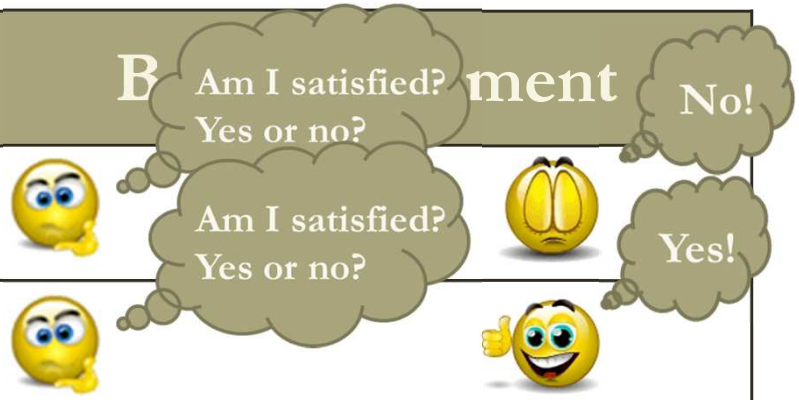
What happens in a NLCUB model?

$T \geq m - 1$ steps in the Feeling phase (for example, let $T = 8$)

Step	B  ment
1	
2	
3	
4	
5	
6	
7	
8	

What happens in a NLCUB model?

$T \geq m - 1$ steps in the Feeling phase (for example, let $T = 8$)

Step	Behavior
1	
2	
3	
4	
5	
6	
7	
8	

What happens in a NLCUB model?

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5	Am I satisfied? Yes or no? Yes!	3
6	Am I satisfied? Yes or no? Yes!	4
7	Am I satisfied? Yes or no? Yes!	5
8	Am I satisfied? Yes or no? Yes!	6

What happens in a NLCUB model?

$T \geq m - 1$ steps in the Feeling phase (for example, let $T = 8$)

Step	B	Accumulated judgement (# of Yes)
1	Am I satisfied? Yes or no? / No!	0
2	Am I satisfied? Yes or no? / Yes!	1
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4	Am I satisfied? Yes or no? / Yes!	2
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6	Am I satisfied? Yes or no? / Yes!	4
7	Am I satisfied? Yes or no? / Yes!	5
8		6

ment

rule

Provisional rating
1
2
2
2
3
3
3
3

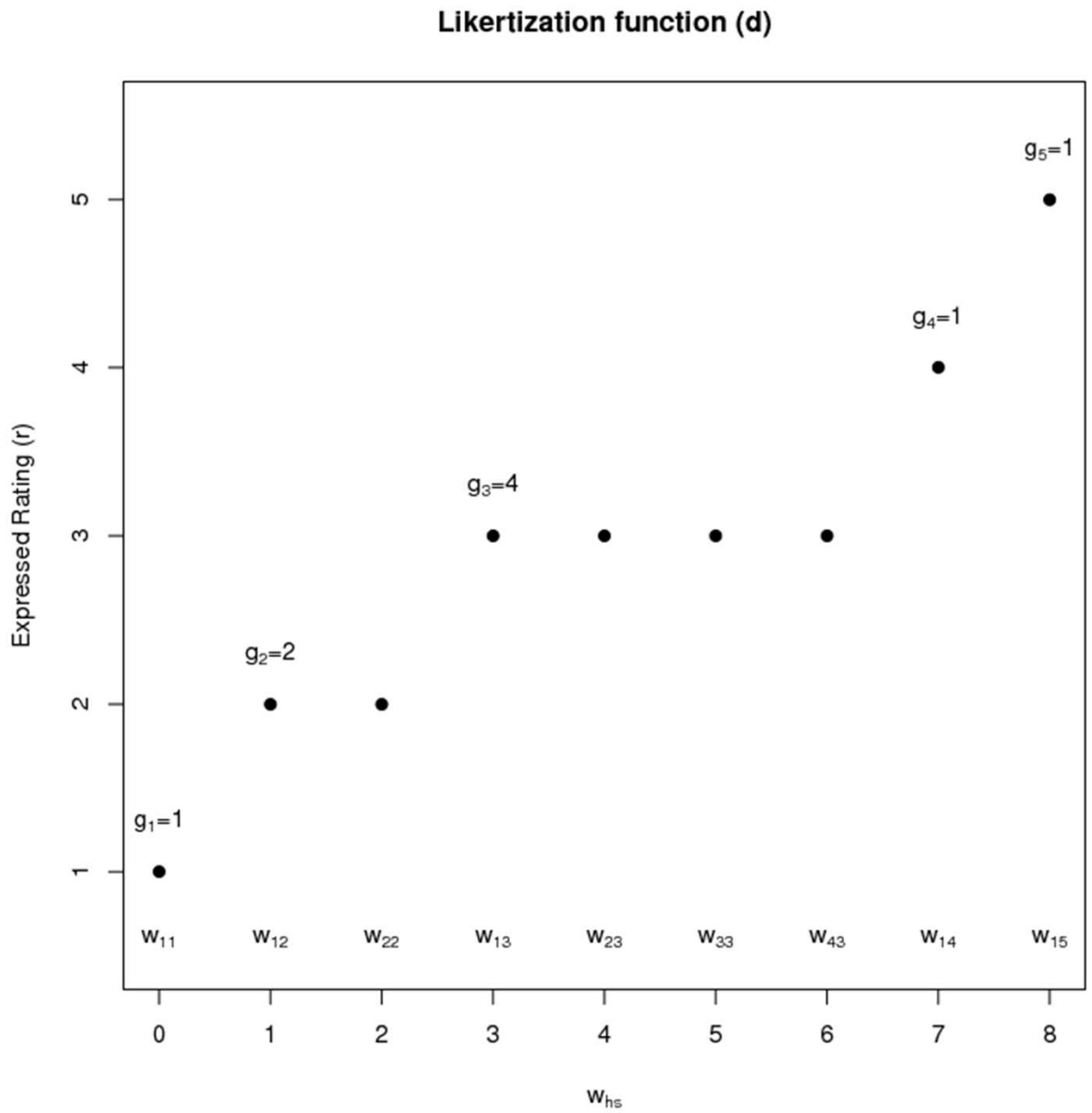
What happens in a NLCUB model?

$T \geq m - 1$ steps in the Feeling phase (f = 8)

Accumulated judgement (# of Yes)	Provisional rating
0	1
1	2
2	
3	3
4	
5	
6	
7	4
8	5

What happens in a NLCUB model?

$T \geq m - 1$ steps in the Feeling phase (for example, let $T = 8$)



The Feeling phase - NLCUB

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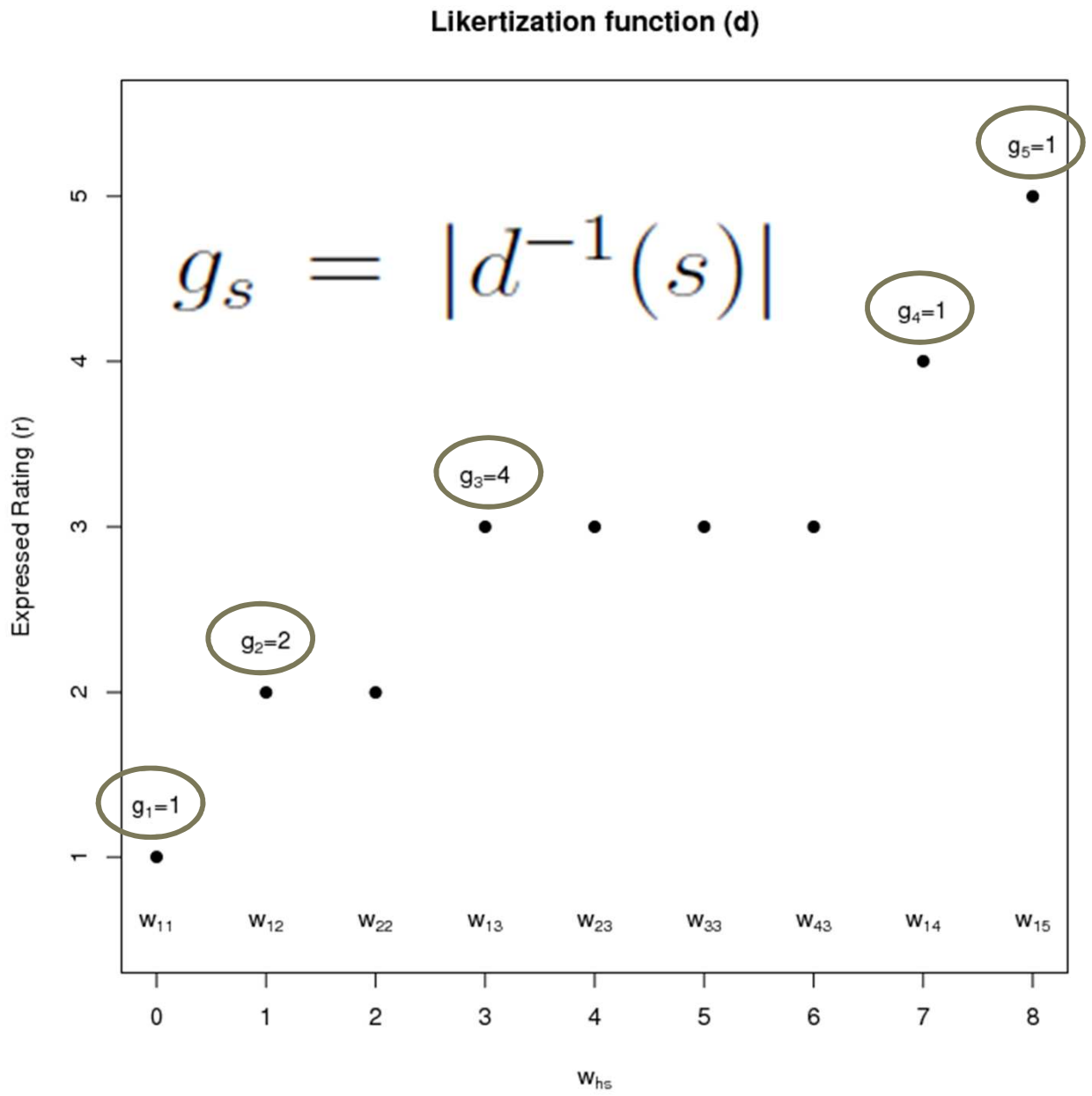

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$T \geq m-1$ steps in the Feeling phase (for example, let $T = 8$)

TRANSITION PROBABILITIES

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

R_t
1
2
2
2
3
3
3
3

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$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

In NLCUB models:

$$\phi_t(s) = (1-\xi)\frac{\binom{t}{w_{g_s s}}(1-\xi)^{w_{g_s s}}\xi^{t-w_{g_s s}}}{\sum_{h=1}^{g_s}\binom{t}{w_{hs}}(1-\xi)^{w_{hs}}\xi^{t-w_{hs}}}$$

R_t
1
2
2
2
3
3
3
3

What happens in a NLCUB model?

$T \geq m-1$ steps in the Feeling phase (for example, let $T = 8$)

TRANSITION PROBABILITIES

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

In NLCUB models:

Different values for different t and s

R_t
1
2
2
2
3
3
3
3

TRANSITION PROBABILITIES

$$\phi_t(s) = Pr(R_{t+1} = s+1 | R_t = s)$$

$$\boxed{\phi(s) = av_t(\phi_t(s))}$$

"Perceived closeness" between rating s and $s+1$

$$\delta_s = h(\phi(s)) \quad \text{for example} \quad \delta_s = -\log(\phi(s))$$

"Perceived distance" between rating s and $s+1$

The transition probabilities

TRANSITION PLOT

A broken line joining points $(s, \tilde{\phi}(s))$, where

$$s = 0, \cdots, m-1$$

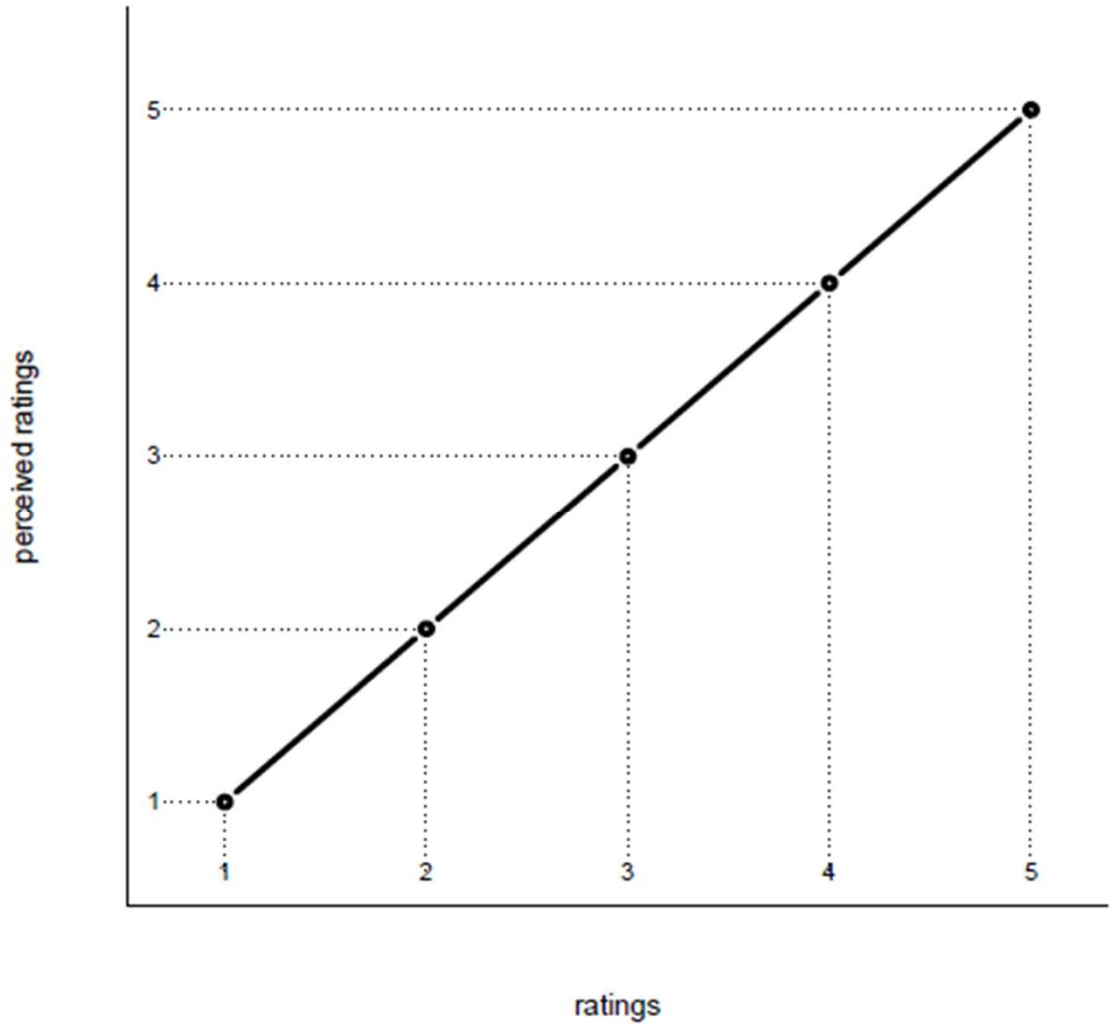
$$\tilde{\phi}(s) = (\delta_1 + \cdots + \delta_s)/(\delta_1 + \cdots + \delta_{m-1})$$

i.e.: the **cumulated** "perceived distances".

It gives an idea of the state of mind of respondents toward the rating scale.

TRANSITION PLOT – CUB model

$$\phi_t(s) = 1 - \xi$$



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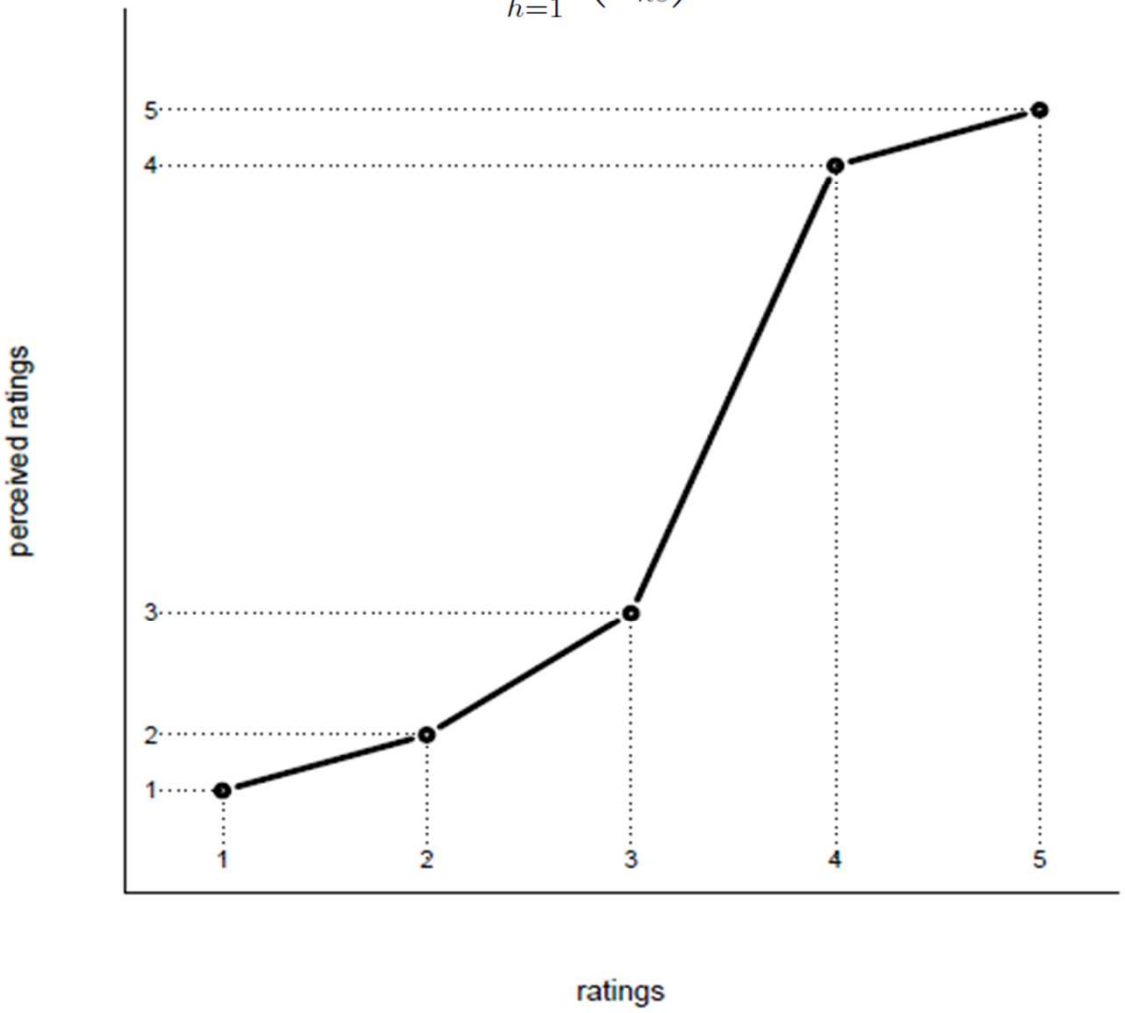

The transition plot

LINEAR DECISION PROCESS

DEFINITION: A DP is said to be linear if $\phi_{t_1}(s_1) = \phi_{t_2}(s_2) = \phi, \forall s_1, s_2, t_1, t_2 : \exists\ \phi_{t_1}(s_1), \phi_{t_2}(s_2)$.

- $\phi_t(s)$: all equal $\leftrightarrow$ **LINEAR DP**
- $\phi_t(s)$: different values $\leftrightarrow$ **NONLINEAR DP**

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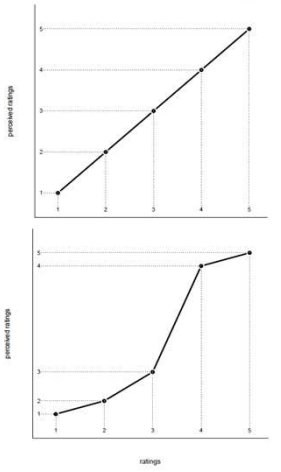
- $\phi_t(s)$: all equal $\leftrightarrow$ **LINEAR DP**
- $\phi_t(s)$: different values $\leftrightarrow$ **NONLINEAR DP**

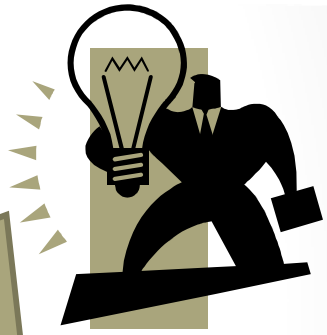
- **LINEAR DP** $\rightarrow$ Linear transition plot
- **NONLINEAR DP** $\rightarrow$ Nonlinear transition plot



Linear vs. Nonlinear DPs

LINEAR DECISION PROCESS

PROPOSITION: A sufficient condition for a discrete additive DP to be linear is that $\forall s$, $d^{-1}(s)$ is composed by a single value and $d^{-1}(1), \cdots, d^{-1}(m)$ are equally spaced in $\mathbb{R}$, that is $d^{-1}(s+1) - d^{-1}(s) = k$ for all s.



The condition is not necessary (stylized facts from a large exploratory study)

NonLinear CUB models

- Derive from a different assumed mechanism in the Feeling phase (the other phases are unchanged)
- Allow us to model nonlinear DPs, gaining insight about the state of mind toward the rating scale
- Include traditional CUB models as a special case

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Are you satisfied with XYZ?

Express a rating from 1 (=completely unsatisfied) to 5 (=completely satisfied)

Feeling phase NLCUB: a random variable (A)

$$P(A=r)=\sum_{y\in l^{-1}(r)} Pr\{V(T+1,\xi)=y\}$$

Uncertainty phase CUB: Uniform random variable (U)

$$P(U=r)=1/m$$

Expressed rating CUB: mixture of V and U (R)

$$P(R=r\mid\boldsymbol{\theta})=\pi\,P(A=r)+(1-\pi)\,P(U=r)$$

NLCUB

Likelihood function

$$L(\boldsymbol{\theta}|\mathbf{s}) = \sum_{i=1}^{n} \log \left\{ \pi \left[\sum_{h=1}^{g_{s_i}} \binom{T}{w_{hs_i}} (1-\xi)^{w_{hs_i}} \xi^{T-w_{hs_i}} \right] + (1-\pi)\frac{1}{m} \right\}$$

(to be maximized via likelihood profiling)

We have to fix a maximum value for T

Likelihood function

$$L(\boldsymbol{\theta}|\mathbf{s}) = \sum_{i=1}^{n} \log \left\{ \pi \left[\sum_{h=1}^{g_{s_i}} \binom{T}{w_{hs_i}} (1-\xi)^{w_{hs_i}} \xi^{T-w_{hs_i}} \right] + (1-\pi)\frac{1}{m} \right\}$$

A good choice may be $T_{max} = 2m\text{-}1$ (stylized facts from a large exploratory study)

NLCUB

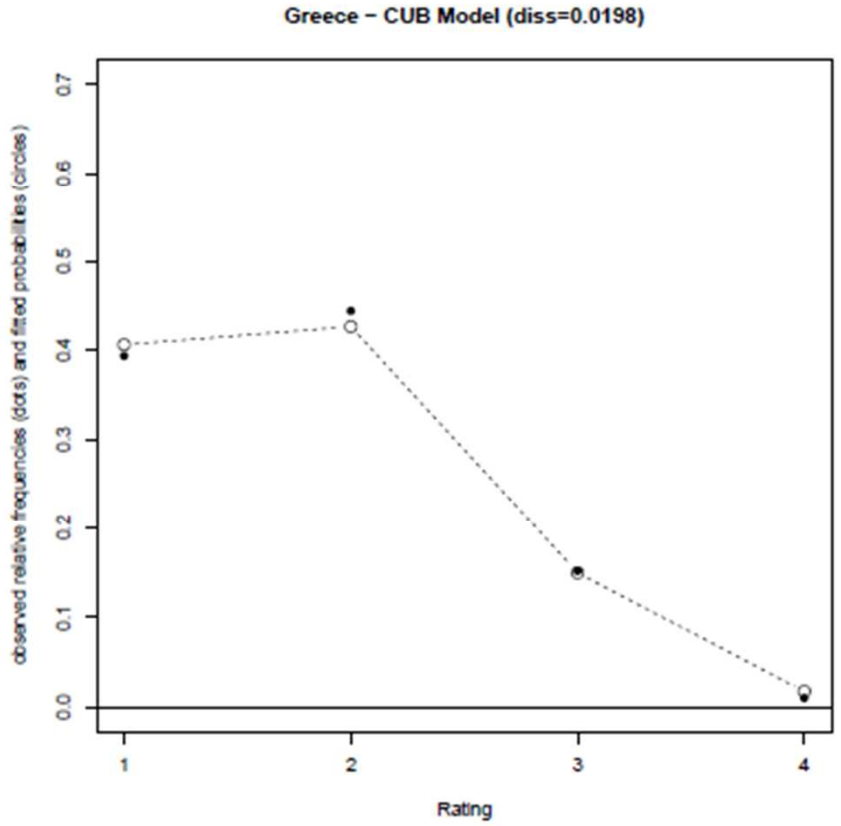
Case study:

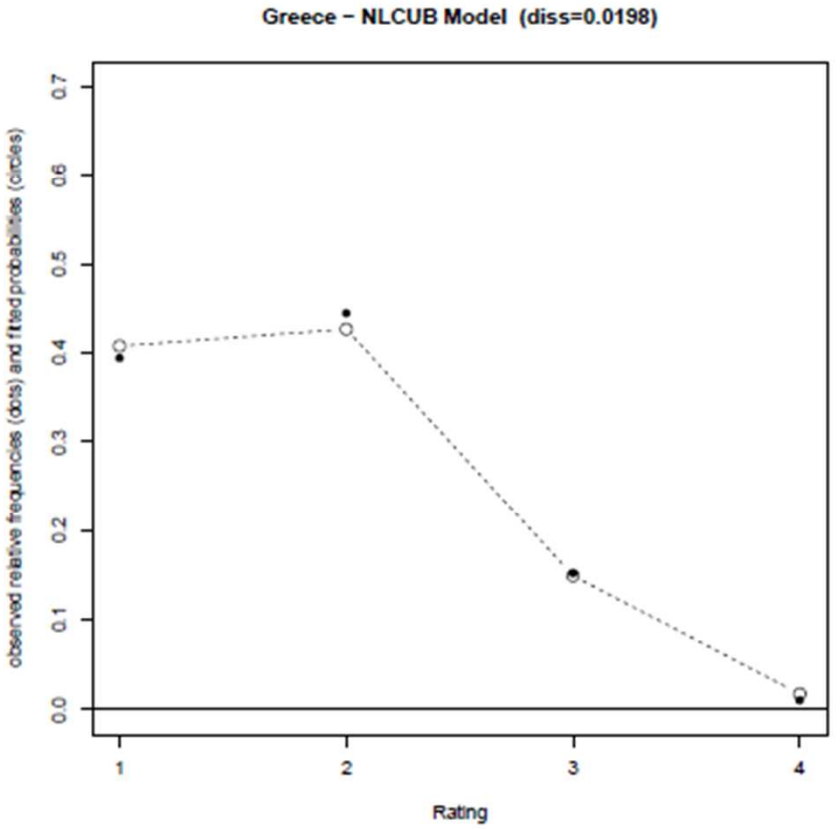
Standard Eurobarometer 78

SYRTO: "SYstemic Risk TOmography: Signals, Measurements, Transmission Channels, and Policy Interventions", grant from the European Union Seventh Framework Programme - Project ID: 320270

- A sample survey covering the national population of citizens of the 27 European Union Member States
- QA3.2: "How would you judge the current situation of the European economy?" (very bad, rather bad, rather good, very good)
- We present results for Greece, Germany, Italy

Greece	ξ	π	g_1	g_2	g_3	g_4
NLCUB	0.7413	0.9999	1	1	1	1
CUB	0.7413	0.9999	1*	1*	1*	1*
		$\phi(1)$	$\phi(2)$	$\phi(3)$	ϕ	μ
NLCUB		0.2587	0.2587	0.2587	0.2587	0.7761
CUB		0.2587	0.2587	0.2587	0.2587	0.7761





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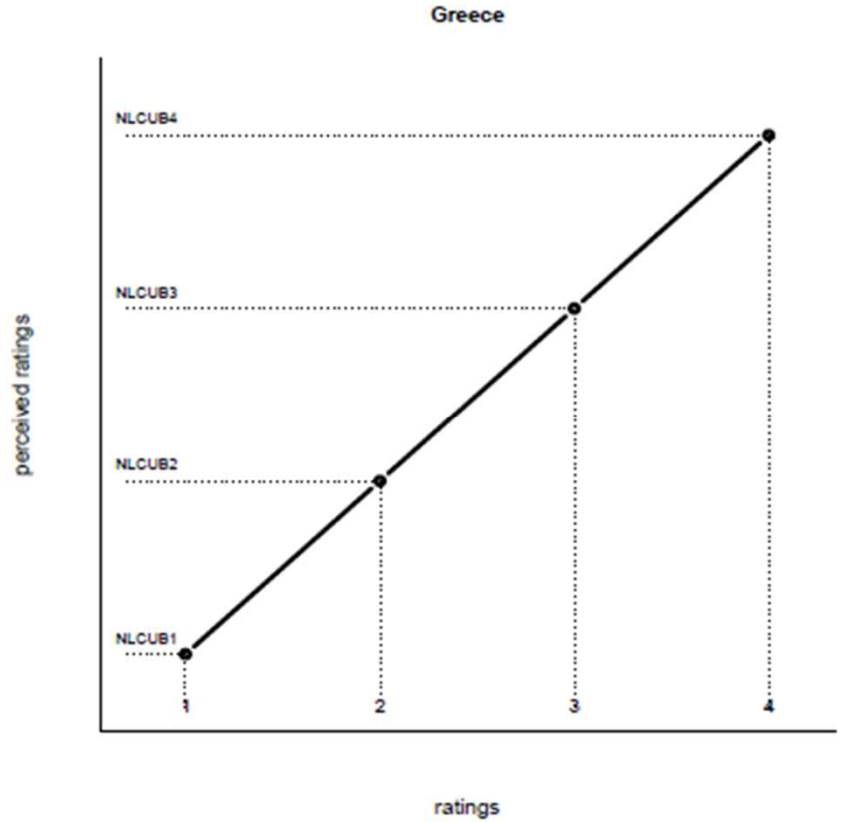
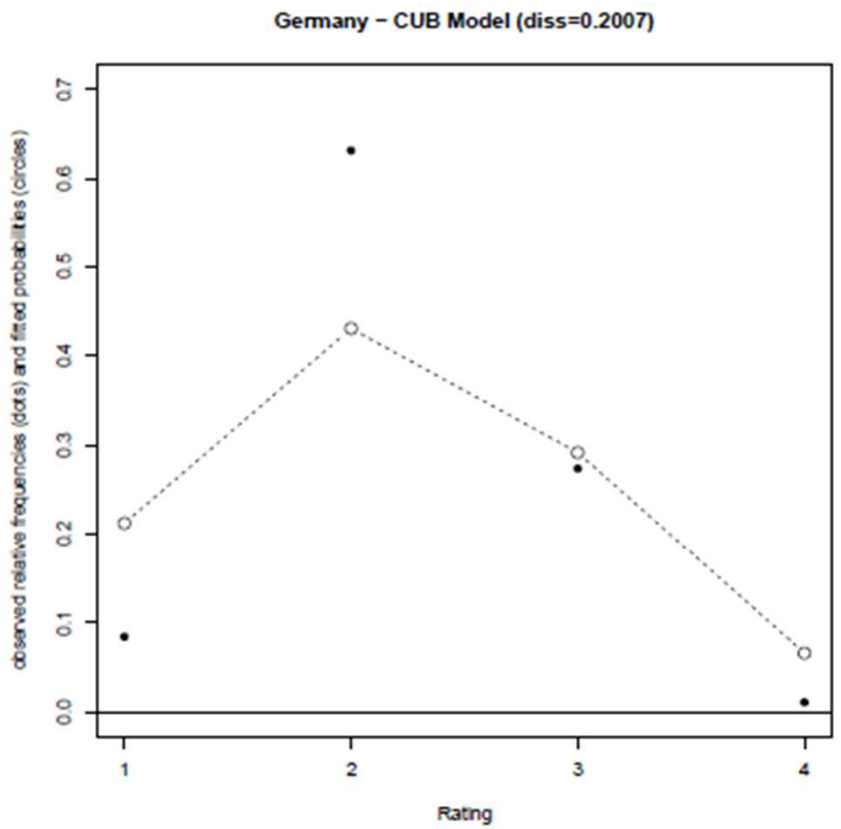
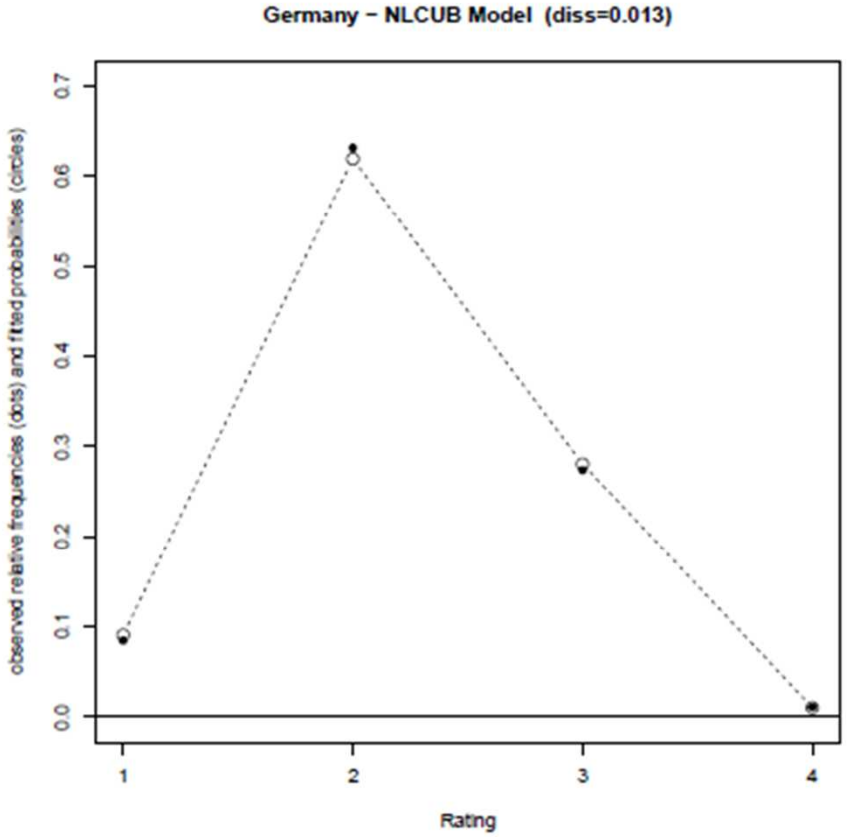


Germany	ξ	π	g_1	g_2	g_3	g_4
NLCUB	0.6167	0.9925	1	2	2	1
CUB	0.5964	0.9999	1*	1*	1*	1*
		$\phi(1)$	$\phi(2)$	$\phi(3)$	ϕ	μ
NLCUB		0.3833	0.1057	0.0258	0.2416	1.2080
CUB		0.4036	0.4036	0.4036	0.4036	1.2108





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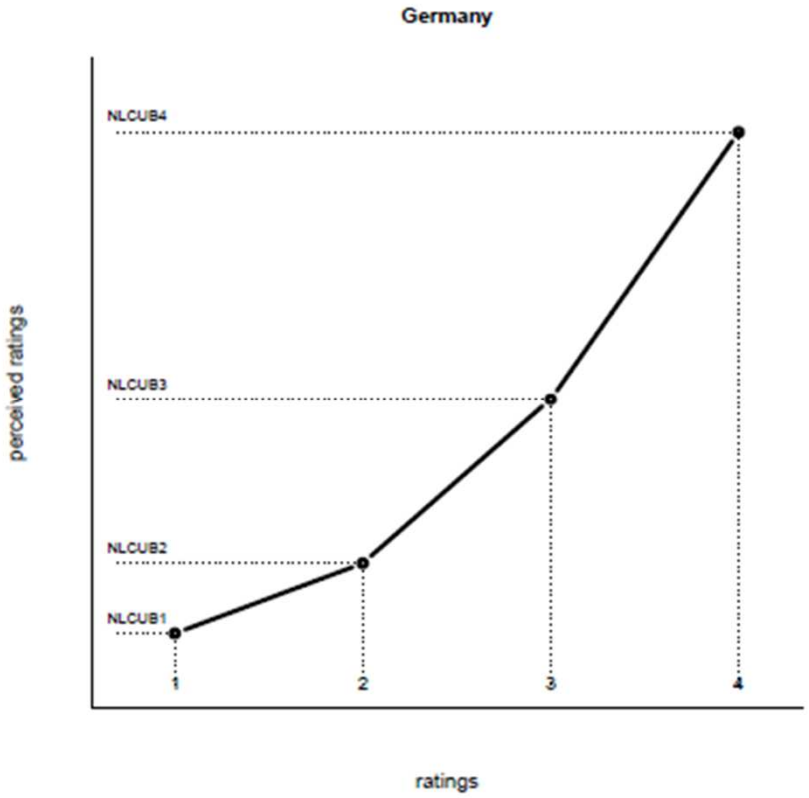
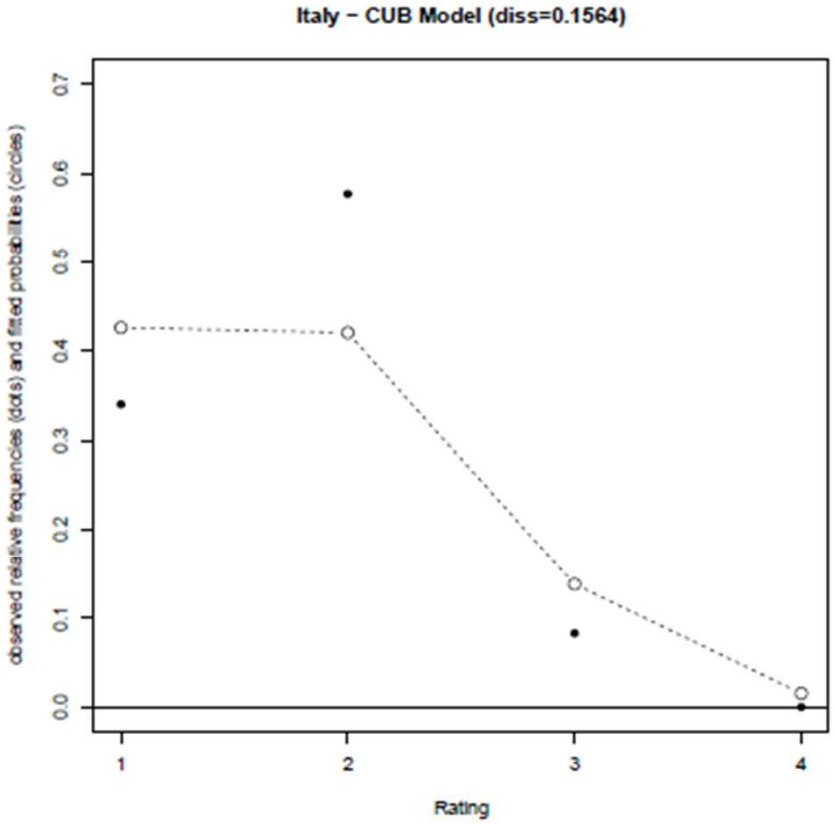
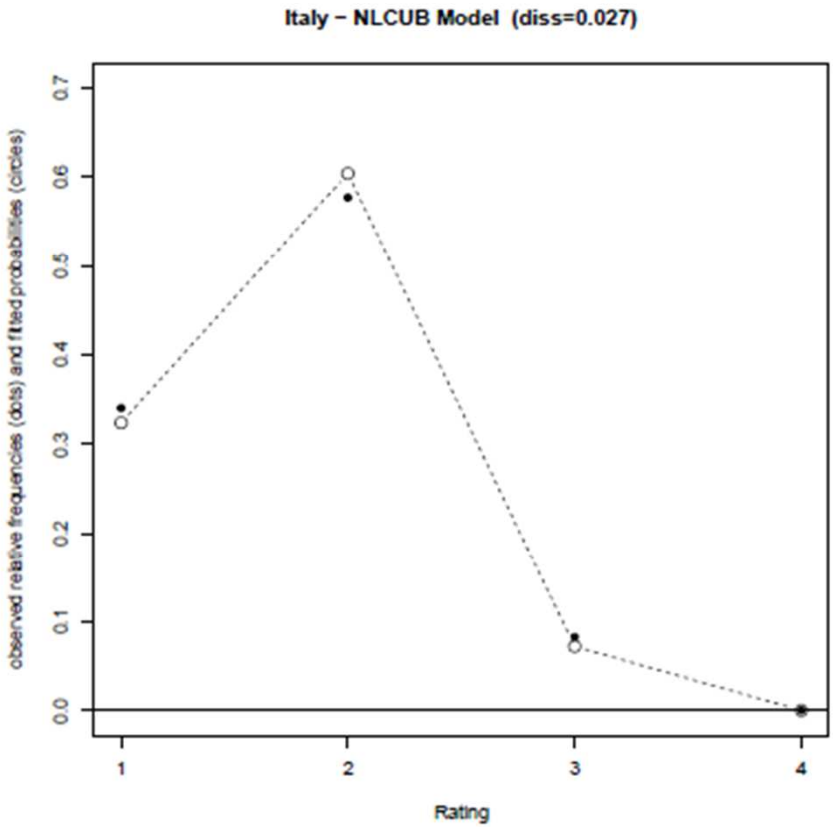


Italy	ξ	π	g_1	g_2	g_3	g_4
NLCUB	0.8512	0.9999	1	2	4	1
CUB	0.7525	0.9999	1*	1*	1*	1*
		$\phi(1)$	$\phi(2)$	$\phi(3)$	ϕ	μ
NLCUB		0.1487	0.0249	8.7e-06	0.1069	0.7484
CUB		0.2475	0.2475	0.2475	0.2475	0.7425





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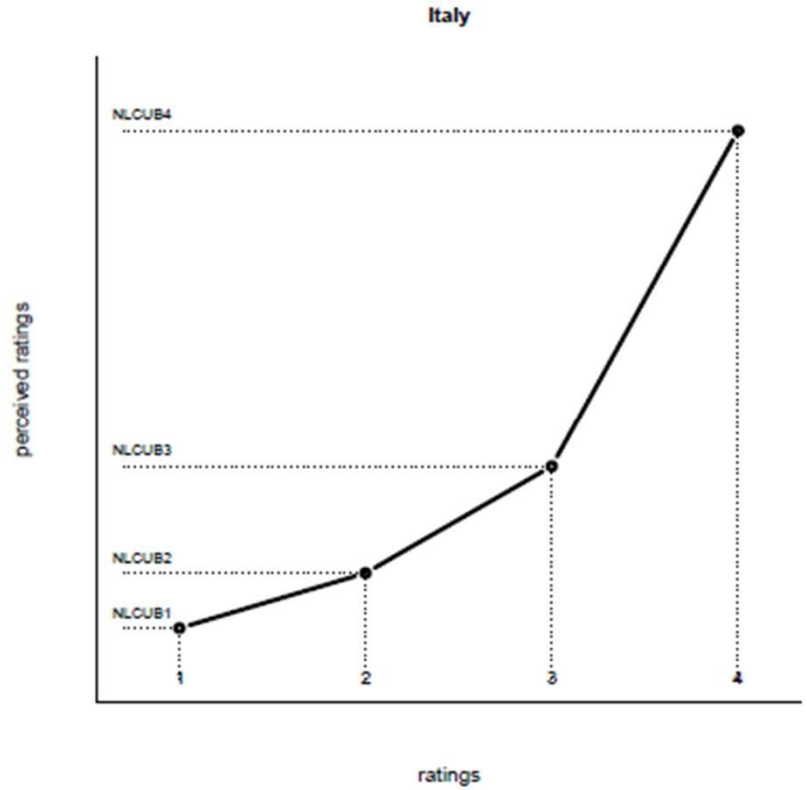

Case study:

Standard Eurobarometer 78

SYRTO: "SYstemic Risk TOmography: Signals, Measurements, Transmission Channels, and Policy Interventions", grant from the European Union Seventh Framework Programme - Project ID: 320270

- All respondents from the three Countries exhibit a very low uncertainty
- Some differences can be observed from the feeling point of view: Italian respondents are the most pessimistic about European economy, immediately followed by Greek respondents. German people, instead, are more confident.

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- Greece shows a linear transition plot: in general, for Greek respondents to move, for instance, from rating 1 to 2 is as hard as from rating 3 to 4.
- Instead, German and Italian respondents find it easier to move from rating 1 to 2 than from rating 3 to 4.

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Basic References

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Thank you for your attention