# Identifying Extreme Representatives in Sports. An Archetypal Approach

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Universitat Politècnica de Catalunya - BarcelonaTech, Spain

Workshop in Advances in Sports Analytics
Statistical Methods, Data Science, and Research Practices
Università degli Studi di Brescia
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# Team work





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National Institute of Physical Education of Catalonia (INEFC)

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# Outline

# Motivation

# Motivation example

**NBA Player Statistics Season 2024-25**

**$n = 501$ players**

**$p = 17$ variables**

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| **MIN** | **Total minutes played** |
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| AST | Assists per game |
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Head of the NBA dataset (some variables)

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- Are they players determined by ADA **representative**? e.g. do they have play enough minutes? Our proposal: **Penalized ADA**.

# Background: AA & ADA

# What are the Archetypes?

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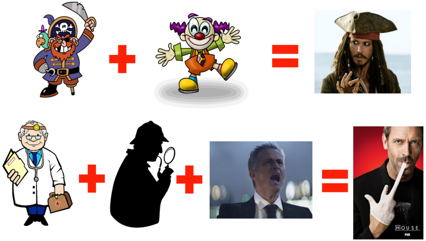

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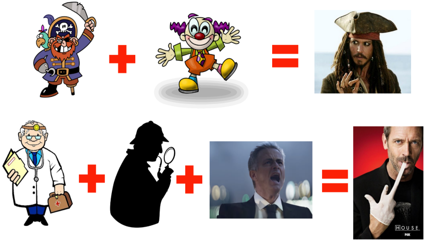


- In Statistics, the **concept of archetypes** is the same as in common life.

# Archetype Analysis (AA). Definition

- **Objective** (Cutler and Breiman, 1994): to find a few, not necessarily observed, extremal cases or pure types (the archetypes) such that:
  - all the observations are **obtained by convex combinations of the archetypes**, and
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- **Solution** (Cutler and Breiman, 1994): Alternating minimizing algorithm and implemented in R by Eugster and Leisch (2009).

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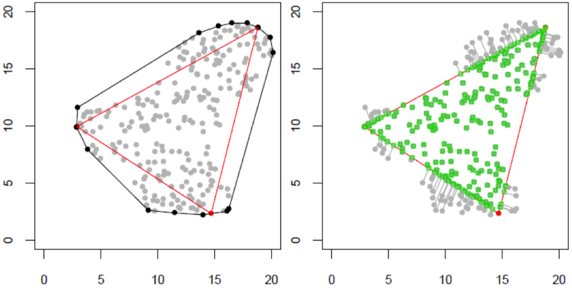


Figure 2: Visualization of three archetypes.

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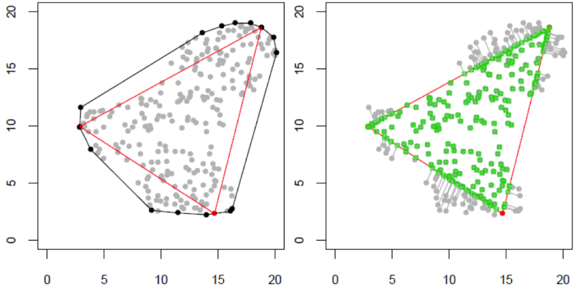


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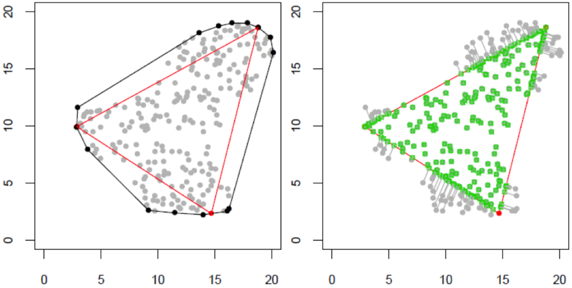


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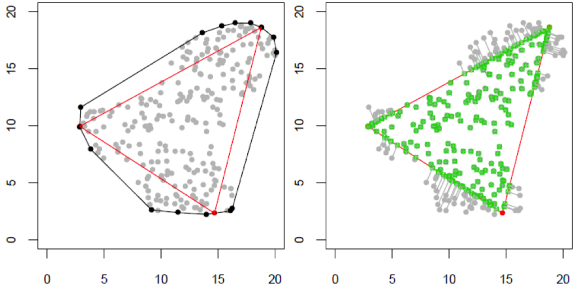


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4. The **observations** are approximated by convex combinations of the archetypes.

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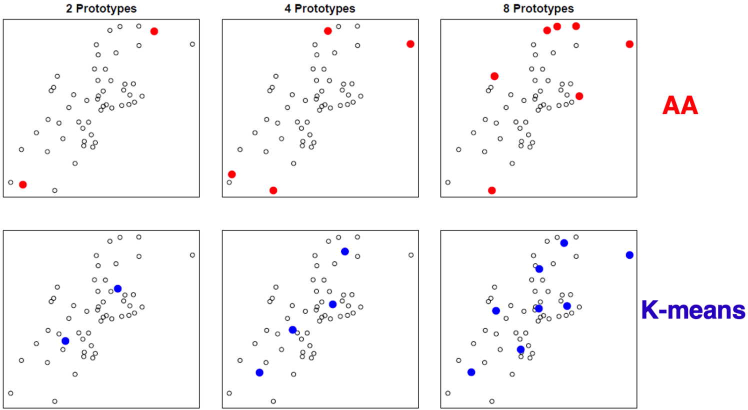
Archetypes **are more interpretable** than the centroids of k-means.



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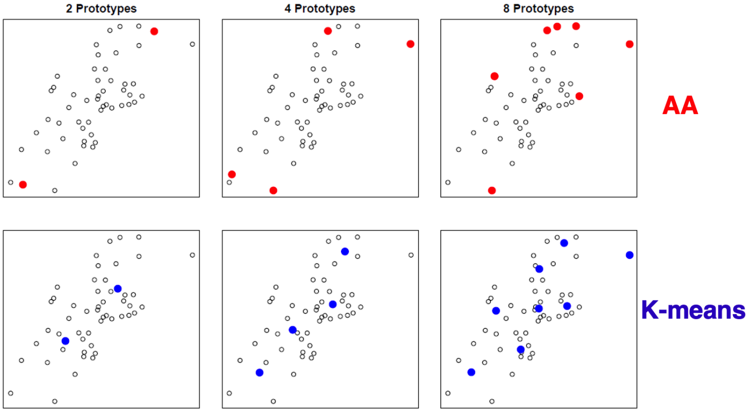
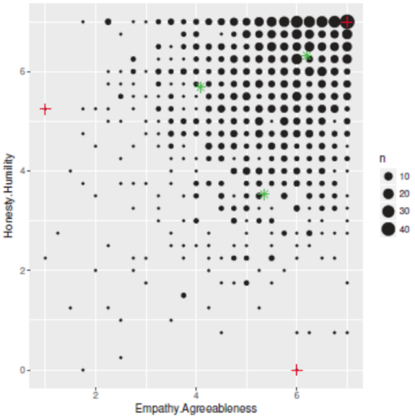



For humans, it is **easier to understand opposites than centroids**: they provide **an intuitive understanding** of the results (Vinúe *et al.*, 2015; Thurau *et al.*, 2012).

# AA. Interpretation

For example, consider two variables: empathy and honesty.

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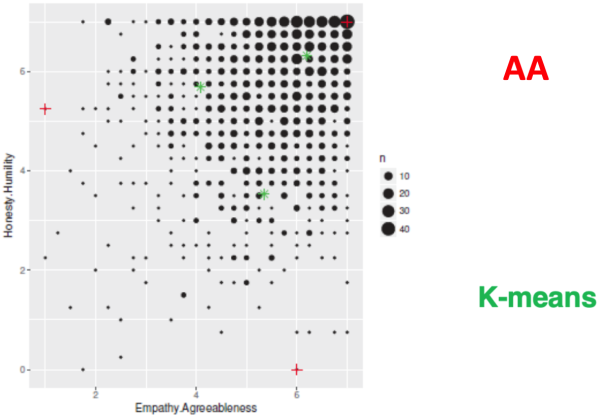


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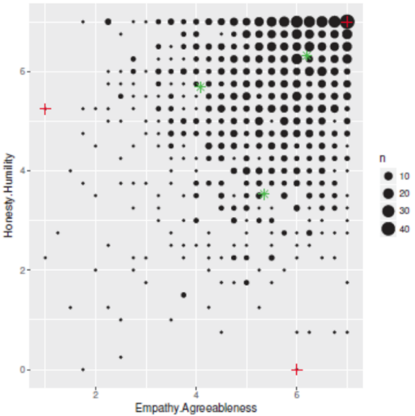


AA

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- **AA**: 3 archetypes:
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- **k-means**:
  - Centroids are in the **middle of the cloud**, not far apart. This method attempts to find **separate groups** in **continuum** data.

# AA for multivariate data

- Let $X$ be an $n \times m$ matrix with $n$ observations and $m$ variables.
- The objective of AA is to find the matrix $Z$ of $k$ $m$-dimensional archetypes.
- AA computes two matrices $\alpha$ and $\beta$ which **minimize the residual sum of squares (RSS)**:

$$RSS = \sum_{i=1}^{n} \|\mathbf{x}_i - \sum_{j=1}^{k} \alpha_{ij}\mathbf{z}_j\|^2 = \sum_{i=1}^{n} \|\mathbf{x}_i - \sum_{j=1}^{k} \alpha_{ij} \sum_{l=1}^{n} \beta_{jl}\mathbf{x}_l\|^2,$$

under the constraints

1) $\sum_{j=1}^{k} \alpha_{ij} = 1$ with $\alpha_{ij} \geq 0$ for $i = 1, \ldots, n$ and

2) $\sum_{l=1}^{n} \beta_{jl} = 1$ with $\beta_{jl} \geq 0$ for $j = 1, \ldots, k$.

The elbow criterion is used to select the number of archetypes.

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  - All archetypoids are **real observations**.

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- **ADA solution** (Vinué, Epifanio, & Alemany, 2015): algorithm based on PAM.

# Summary of main developments in AA and ADA.

1. **Research**
   - **Functional AA (FAA) and functional ADA (FADA)** (Epifanio, 2016): to find k archetype functions (mixture of the data) or to find k functions of the sample (archetypoids).
   - **Outliers** (Vinúe & Epifanio (2021); Cabero, Epifanio *et al.* , (2021)): Detecting outliers with AA and ADA.
   - **Binary data** (Cabero & Epifanio, 2020).
   - **Ordinal data** (Fernández, Epifanio, & McMillan 2022).
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2. **R packages**:
   - archetypes (Eugster and Leisch, 2009)
   - adamethods (Vinué & Epifanio, 2021)
   - Anthropometry (Vinué and Epifanio, 2021)

# Illustration: ATP data

INTERNATIONAL JOURNAL OF SPORTS PHYSIOLOGY AND PERFORMANCE

**Identifying Extreme Representative Tennis Players and Match External Load in Male Grand Slam**

| | |
|---|---|
| Journal: | *International Journal of Sports Physiology and Performance* |
| Manuscript ID | IJSPP.2024-0532 |
| Manuscript Type: | Original Investigation |
| Date Submitted by the Author: | 09-Dec-2024 |
| Complete List of Authors: | Brich, Quim; Institut Nacional d'Educació Física de Catalunya,<br>Casals, Martí; National Institute of Physical Education of Catalonia; Universitat de Vic - Universitat Central de Catalunya, Sport and Physical Studies Centre (CEEAF)<br>Cortés, Jordi; Universitat Politècnica de Catalunya, Department of Statistics and Operations Research<br>Fernández, Daniel; Universitat Politecnica de Catalunya, Department of Statistics and Operations Research<br>Baiget, Ernest; National Institute of Physical Education of Catalonia, |
| Keywords: | Workload, Multivariate Data, Archetypoid Analysis, Unsupervised Learning, Professional Players |
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# ATP data: Dataset Description

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- **Objective**: **Player Profiling** $\Rightarrow$ to identify **real extreme** player profiles (**ADA**).

# ATP data: ADA results (players)

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- **Cluster 3 (34.2%)**: **Low volume, moderate-high intensity** archetype; players with efficient, high-impact performance. *Representative: Luca Vanni*

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- Each player who participated in a Grand Slam in 2017 is represented as a mixture of the four archetypoids .

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- **Cluster 4 (21.1%)**: Highest volumes with **moderate-low intensity**; endurance-based style with steady output.
  *Representative: Janko Tipsarevic*

## ATP data: ADA results (players)

# Darian King


Sam Groth, Luca Vanni, and Janko Tipsarevic

# Proposed method: Penalized ADA

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  2. The **penalty weights** are defined as:
     - $w_\ell^0 = 1/(\mathrm{minutes}_\ell + \epsilon)$, where $\epsilon$ is a small constant to avoid division by zero,
     - $w_\ell = w_\ell^0 / \bar{w}^0$, where $\bar{w}^0$ is the mean of all the $w_\ell^0$,
     - $\lambda \geq 0$ controls the strength of the penalty; $\lambda = 0$ represents no penalization.
- **Interpretation**: High-minute players $\rightarrow$ low contribution to the penalty

# Application: NBA data set

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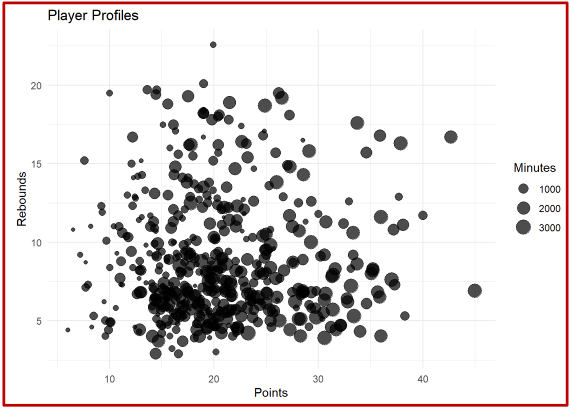
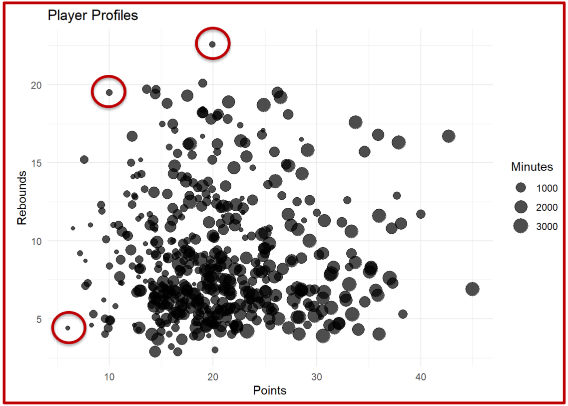



- The variables **Points** & **Rebounds** are our features of interest.
- The variable **Minutes** serves to compute weights.

# Standard ADA




- The variables **Points** & **Rebounds** are our features of interest.
- The variable **Minutes** serves to compute weights.
- Note that some players with very **low minutes** are located at the **extremes** of the data cloud $\Rightarrow$ Archetypoids in the **standard ADA**.

# ADA results: Standard vs. Penalized (I)

- Example: $k = 3$ archetypoids, 20 starting points, & $\lambda = 1$ as the penalty strength.

| | **Standard** | **Penalized** |
|---|---|---|
| RSS | 0.027 | 0.159 |
| Avg. minutes | 988.97 | 2421.67 |
| Computation time | 27.95 | 41.11 |

- **RSS** is higher in the penalized version $\Rightarrow$ adding the penalty selects **less extreme** but more **representative** archetypoids.
- **Average minutes** nearly triples in the penalized version (989 $\rightarrow$ 2422 min) $\Rightarrow$ confirms the penalty favors **high-minute** players.

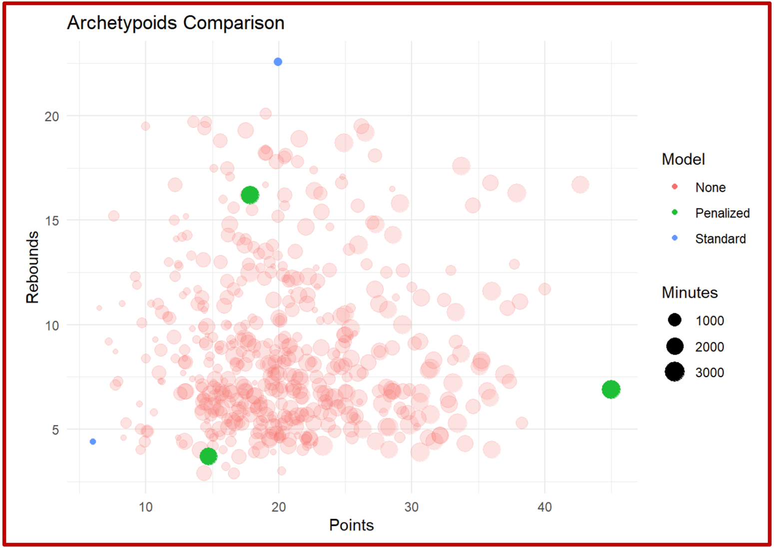
# ADA results: Standard vs. Penalized (II)

**Selected archetypoids**

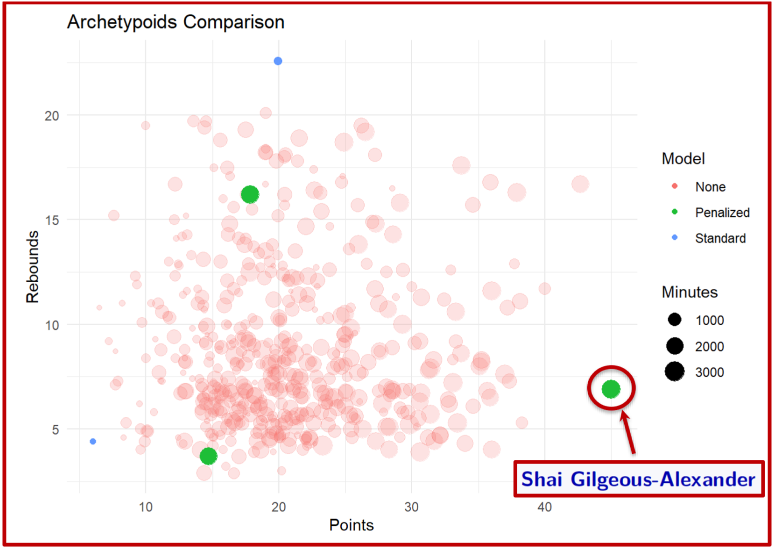
| Player | PTS | REB | MIN | Model |
|---|---|---|---|---|
| Joe Ingles | 6.0 | 4.4 | 114.1 | Standard |
| Oscar Tshiebwe | 19.9 | 22.6 | 255.2 | Standard |
| **Shai Gilgeous-Alexander** | 45.0 | 6.9 | 2597.6 | **Both** |
| Rudy Gobert | 17.8 | 16.2 | 2388.4 | Penalized |
| Kentavious Caldwell-Pope | 14.7 | 3.7 | 2279.0 | Penalized |

- Standard ADA picks players with **very low minutes** (Joe Ingles: 114 min, Oscar Tshiebwe: 255 min), which may be **misleading** in practice.
- **Shai Gilgeous-Alexander** is selected in **both** versions: he is simultaneously **extreme and representative**.
- $\lambda \geq 0$ controls the trade-off: higher $\lambda$ favors **representativeness**, lower $\lambda$ favors **extremeness**.

# ADA results: Standard vs. Penalized (III)



# ADA results: Standard vs. Penalized (III)



# Penalized ADA results (players)

- Each NBA player is represented as a mixture of the 3 archetypoids.
  Example Cluster 3:

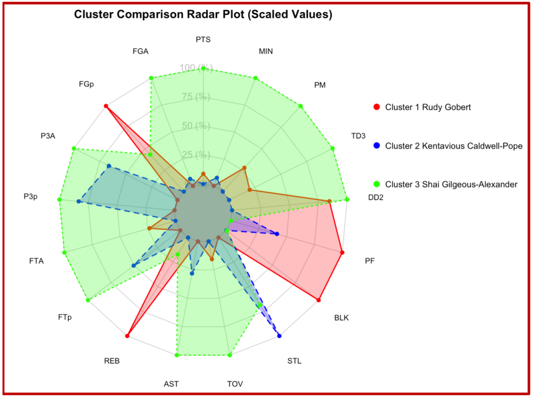
**Cluster 3 (N=58; 11.6%)**


***Representative: Shai Gilgeous-Alexander***

- Point guard - Shooting guard.
- 4-time NBA All-Star.
- 3-time All-NBA First Team member.
- 2-time NBA MVP.

# Penalized ADA results (players)

## Cluster 3 (N=58; 11.6%). Representative: Shai Gilgeous-Alexander




- **Cluster 3** players are **elite, high-usage franchise stars who dominate offensively**.

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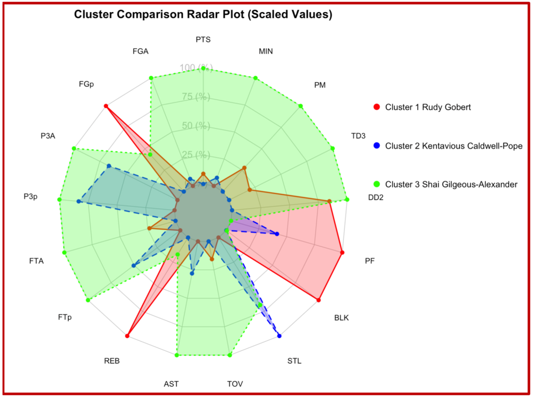


- **Cluster 3** players are **elite, high-usage franchise stars who dominate offensively**.
- **Leading all clusters** in scoring (32.9 pts), assists (7.22), minutes (1,857), and free throw attempts (7.57).

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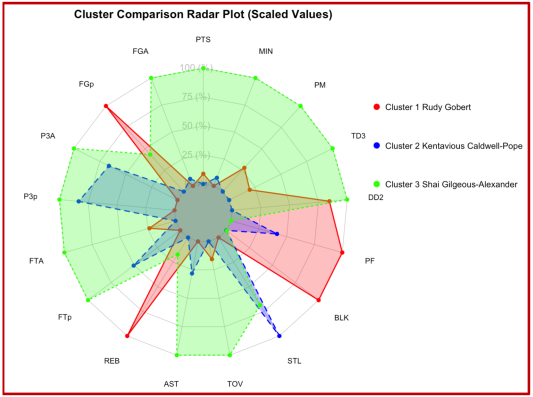


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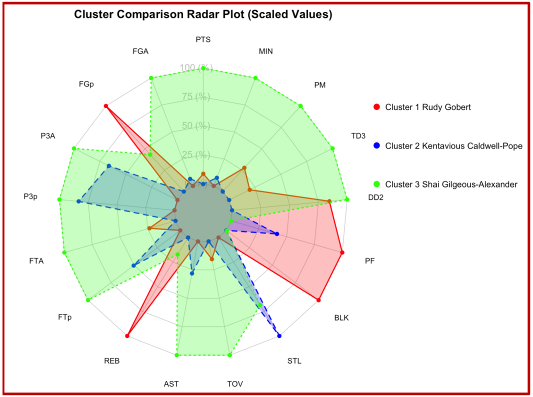


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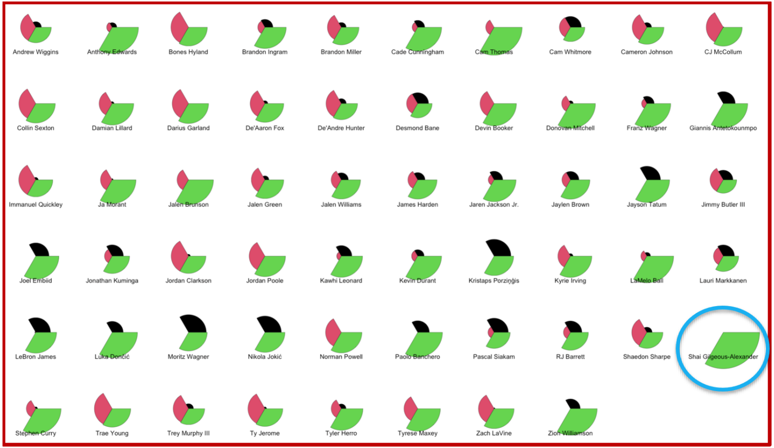
- They are the only cluster with a positive plus/minus (+1.37), confirming their **winning impact**.
- Though their heavy playmaking responsibility comes at the cost of the **highest turnover rate** (3.77).

## Shai Gilgeous-Alexander (green archetypoid)



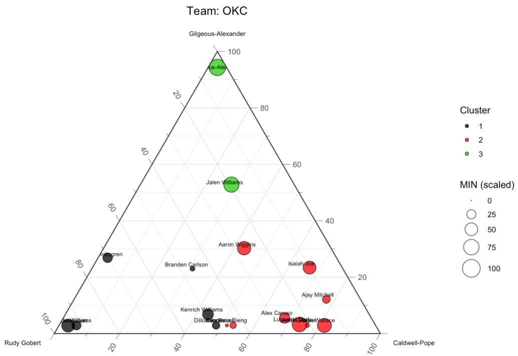

Rudy Gobert, and Kentavious Caldwell-Pope

# Penalized ADA results (players)

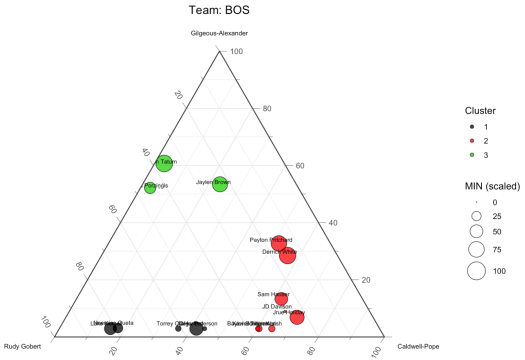
## Shai Gilgeous-Alexander (green archetypoid)


Rudy Gobert, and Kentavious Caldwell-Pope

# Penalized ADA results (teams) - playing style




OKC's simplex is dominated by two star players pulling the team **toward the extremes (ADAs)**.




Boston's players are spread more broadly through the **interior of the triangle,** consistent with a deeper rotation and a more collectively **balanced playing style**.

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- In **sports analytics**, where real-world relevance is essential, penalized ADA provides a more **actionable** solution than standard ADA.

# Next Steps

### 1 Sensitivity Analysis in $\lambda$

How the penalty **parameter $\lambda$** affects the selection of archetypoids.

### 2 Simulation Study

Simulation experiments to evaluate the method under **known ground truth conditions**.

### 3 Radial Approach

**Alternative ways** of incorporating representativeness via a radial approach for selecting archetypoids.

### 4 R Package

**R package** implementing the proposed methodology.

# Main References of this Talk


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- Vinúe, G., and Epifanio, I. (2021). Robust archetypoids for anomaly detection in big functional data. *Advances in Data Analysis and Classification*, 15, 437-462.

# The End

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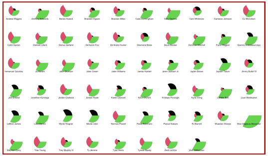


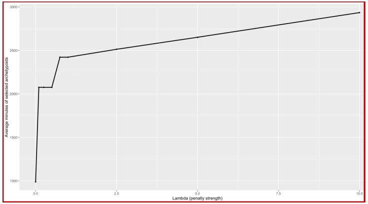
# Extra

# Influence of the Penalty ($\lambda$)

- For fixed $k = 3$, we compute the **average minutes** of the selected archetypoids across a range of $\lambda$ values.
- As $\lambda$ increases:
  1. **Avg. minutes** $\uparrow \rightarrow$ more representative players.
  2. **RSS** $\uparrow \rightarrow$ less extreme archetypoids.
  3. **Ratio** (min/RSS) $\uparrow \rightarrow$ better balance.
- Large $\lambda$ favors **high-minute** players at the cost of a higher RSS.

Results for varying $\lambda$ ($k = 3$)

| $\lambda$ | Avg. Min | RSS | Ratio |
|---|---|---|---|
| 0.00 | 1238 | 8.69 | 142 |
| 0.10 | 1589 | 8.72 | 182 |
| 0.25 | 1589 | 8.72 | 182 |
| 0.50 | 1589 | 8.72 | 182 |
| 0.75 | 1589 | 8.72 | 182 |
| 1.00 | 1589 | 8.72 | 182 |
| 2.50 | 2063 | 8.92 | 231 |
| 5.00 | 2241 | 9.06 | 247 |
| 10.00 | 2603 | 9.92 | 262 |



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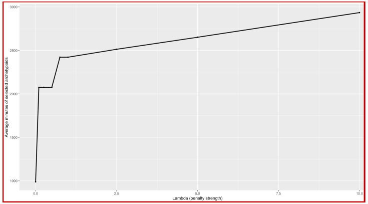

# Interpretation

- As $\lambda$ increases, the model favors players with **higher minutes**, leading to more **representative** archetypoids.
- This illustrates a key **trade-off**:
  1. **Low** $\lambda \rightarrow$ more **extreme** but less reliable archetypoids.
  2. **High** $\lambda \rightarrow$ more **representative** but less extreme.
- A well-chosen $\lambda$ **balances** both aspects.
- The penalized version tends to select players with **higher minutes**, improving **interpretability** and practical relevance.
- In **sports analytics**, where real-world relevance is essential, penalized ADA provides a more **actionable** solution than standard ADA.
- Standard ADA may select **outliers** with few minutes played, leading to **misleading** prototypes.

# Conclusions. Penalized ADA

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